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REVISING R&D PROGRAM BUDGETS WHEN CONSIDERING FUNDING CURTAILMENT WITH A WEIBULL MODEL

THESIS

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AFIT/GAQ/ENS/01M-01

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THESIS

Presented to the Faculty
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In Partial fulfillment of the Requirements for the
Degree of Master of Science of in Acquisition Management

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Paul H. Porter

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We use the superscript *org* for original program profile and *rev* for revised program profile to indicate the data that the variable is based upon. In most equations, we include the superscript in those equations that apply to both the original and the revised programs.

Sets and Indices (superscript *org* of original program and *rev* for revised program):

$n \in N^{org}, N^{rev}$	Year index for N budget years
$j \in J$	Year index for J spendout (outlay) years
$i \in N + J - 1$	Year index for N budget years and J spendout years
t	Time since beginning of program expressed in years
t_i	Time at end of *i*th fiscal year

Data:

B_i	Budget (obligation authority) for year i expressed in current (then-year) dollars
s_j	Spendout or outlay rate (proportion) for year j of a year's budget authority
c_i	Inflation factor for year i for Military Department's R&D expenditures

Variables:

O_i^{org}, O_i^{rev}	Expected outlay or expenditures based on a budget for year i expressed in current (then-year) dollars
$\widetilde{O}_i^{org}, \widetilde{O}_i^{rev}$	Expected outlay or expenditures based on a budget for year i expressed in constant (base-year) dollars
$\hat{O}_i^{org}, \hat{O}_i^{rev}$	Modeled outlays or expenditures for year i expressed in constant dollars
$\hat{B}_i$	Model budget (obligation authority) for year i expressed in current dollars
$F(t)$	Cumulative percent of expenditures at time t for various distributions

$f(t)$	Rate of expenditures at time t for Rayleigh or Weibull models
a	Rayleigh scale parameter
b	Rayleigh shape parameter (if $b \neq 2$ results in Weibull distribution)
D	Total program cost expressed in constant (base-year) dollars
d	Cost factor to scale distributions to dollars
$E(t)$	Modeled cumulative expenditures at time t
t_{peak}	Time of peak rate of expenditures for Rayleigh cost model
t_{final}, t_{final}^{org}, t_{final}^{rev}	Modeled program completion for Rayleigh, original Weibull, and revised Weibull models, respectively
γ	Weibull location parameter
δ^{org}, δ^{rev}	Weibull scale parameter
β^{org}, β^{rev}	Weibull shape parameter

Current means the same as then-year dollars, which are the actual dollars spent at a time. Constant dollars are the same as base-year dollars, which are the dollar values adjusted for inflation. Congress grants programs obligation authority, commonly called budget dollars, in a particular fiscal year. For R&D programs, that obligation authority is spent or obligated to contracts over a period of several years. The spendout or outlay rates reflect the portion of budget authority, expressed in current dollars, that is spent in each year beginning in the fiscal year of the authority. Expenditures and outlays are synonymous terms.

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Abstract

This research develops an analytical technique to estimate the impact of funding curtailment on an R&D program. The method quickly produces a revised budget by year for an on-going R&D program when funding in one year is reduced. We assume program requirements remain unchanged. The program duration may be unchanged or "stretched" to a later completion date. We use the Rayleigh and Weibull functions to model expenditure profiles, which forms basis of the analytical approach. Our proposed methodology accounts for budget outlay rates and inflation. We validate the proposed analytical technique using historical cost data from several programs.

REVISING R&D PROGRAM BUDGETS WHEN CONSIDERING FUNDING CURTAILMENT WITH A WEIBULL MODEL

I. Introduction to the Research

General Issue

The military departments and the Office of the Secretary of Defense (OSD) conduct annual reviews of their programs and budgets. At times, the total program requests exceed prescribed funding goals. Rather than terminate programs, funding levels for development and production programs are often curtailed to comply with spending limitations. The funding reduction lessens the effort on the project in that fiscal year, and may extend the completion date if the program requirements remain unchanged.

For programs in the production phase, budget analysts can easily determine and revise the budget profile. Cost progress curves, sometimes called learning curves, relate the quantity produced in any fiscal year to the required cost. An analyst can reduce annual production quantities until the cost is below funding limits.

For programs in the research and development (R&D) phase, the impact of a budget reduction is less certain. Without a corresponding reduction in program content, the imposed funding constraints generally prevent acquisition objectives and milestones to be reached as scheduled; hence, the timeline is "stretched."

Besides just shifting effort in time, the R&D program likely encounters inefficiencies that increase costs. For example, development of a component may be stopped as part of reducing the effort, but an additional restart cost is incurred when the development is resumed. Another example is a group of engineers may not be as productive because the required interface development is delayed. Although in the short-term money is "saved," financial analyst realize that in the long run the program will incur additional cost more than the amount "saved."

Putman (1978:348) and Gallagher and Lee (1996:52) indicate that there is an efficient rate at which a program in the R&D phase expends funds. Jarvis and Pohl (1999) review three models for expenditure rates. Lee, Hogue, and Gallagher (Lee, 1997:31) report that the Rayleigh Model fit contract expenditures from 20 defense R&D acquisition programs and how to account for outlay rates.

Once money is removed from a program, the funding profile for the program is permanently changed. The changed profile should reflect a different efficient expenditure profile. We depict the initial expenditure and revised expenditure profile with a Weibull model.

Specific Problem

Cost progress (learning) curves provide quick means to estimate revised budgets on production programs. The focus of this research is to develop an analytical model to estimate the impact of funding curtailment on R&D programs. This research develops an approach to assess funding curtailment impact to an R&D program's overall cost and schedule.

Research Benefits

This research provides the military financial communities, such as the Deputy Undersecretary of the Air Force for Financial Management (SAF/FM) and OSD Program Analysis & Evaluation (PA&E), with a method to develop general estimates of total cost with revised budgets when they consider curtailment of funding to a program in the R&D phase.

Other Considerations

The purpose of this research is to provide financial organizations in the military departments and OSD PA&E with a model that can be used quickly to give a good estimate of the effects of the proposed curtailment of funding. The usefulness of the proposed model is the speed at which the impact of the curtailment can be determined. This model is not a substitute for the program office's detailed estimates. Our approach provides a quick approximation while leaving the specifics to the program office.

We also answer other questions related to this topic. When funding is cut to "save" money in one year, can we quantify, in dollars, the impact this action has in the total cost of the program? How much does that "curtailed" dollar "cost" in relation to the program? For example, does curtailing $200 million today add $350 million to the total cost of the R&D program? What is the resulting required budget profile to support an efficient expenditure plan? Can budget analysts use the Rayleigh model to provide revised budget profiles of defense acquisition programs in the research and development phase that have had their funding curtailed in one fiscal year?

This chapter introduces the general problem associated with funding curtailment for a program in the R&D phase. This chapter highlights some of the consequences associated with curtailing program funding. Chapter Two reviews the literature on cost modeling R&D programs. Chapter Two identifies the Rayleigh Cost Model as a valuable tool in forecasting budgets. Chapter Three describes the analytical approach we propose to forecast a budget when funding curtailment is proposed. Chapter Four provides the results of using historical programs as a validation of the methodology outlined in Chapter Three. Finally, Chapter Five presents general conclusions and findings on our proposed methodology.

II. Literature Review and Concept Definition

Chapter Overview

Development of an appropriate funding profile for an R&D program is a key concern for those with a vested interest in the success of the program. If the program is not sufficiently funded, advancement of the program is hindered. If the program is over-funded, money spent on the program could be used elsewhere with little impact to the program itself. Cost analysts employ different mathematical modeling techniques to develop the appropriate funding profiles.

This chapter first presents current mathematical models detailed in the literature. The Rayleigh Model, the final model presented, is explored in depth. The Rayleigh probability density function and cumulative distribution function are presented and graphed. The discussion includes the parameters and variables necessary for the Rayleigh Model to be an accurate forecasting tool. This chapter concludes with an explanation of how the Rayleigh model applies to funding issues.

Large defense R&D programs are complex and take years to complete. The program expenditure requirements needed each year must be met for a program to finish on schedule. A defense program's R&D budget for any particular year is expended or outlayed over several years. The outlay pattern varies little from year to year. The budget must be sufficient to meet expenditures considering the outlay delay. Several models aid in forecasting fiscal requirements for a program in the R&D phase. These models when scaled fit the expenditures over time in constant dollars. Each of these models fit expenditures, which is the budget obligation authority that is spent over J years.

Converting Budget Profiles to Expenditure Profiles

R&D programs in the Department of Defense receive budget authority or total obligation authority (TOA) that may be spent over several years. Since this funding provides the resources necessary to carry out the program over multiple years, the OSD comptroller publishes standard outlay (expenditure) patterns for these programs as part of the annual budget. The outlay pattern is the targeted fixed percentage of funds to be spent each year.

To change from a budget to an expenditure profile, outlay rates are applied to the budget. The result is the expenditure profile in then-year dollars, which are the annual dollar amounts spent on the program. To do this, the formula we use is

$$O_i = B_i s_1 + B_{i-1} s_2 + B_{i-2} s_3 + \ldots + B_{i-J} s_J \qquad (1)$$

with $B_i = 0$ for $i \leq 0$ and where O_i is the expenditures and B_i is the authorized budget amount for *i*th year in then-year dollars, s_i is the outlay rate for *j*th year of the budget, and J is the total number of outlay years.

Then-year dollars have an inflation component built into them. The next step is to remove this inflation factor in the expenditure profile in order to obtain the expenditure profile in base-year dollars. We divide each annual expenditure by the appropriate inflation index factor.

$$\tilde{O}_i = \frac{O_i}{c_i} \qquad (2)$$

where $\widetilde{O}_i$ is the expenditure for ith in base-year dollars, c_i is the inflation factor of the given year, and O_i is the expenditures for ith year in then-year dollars. This step obtains an expenditure profile of the program in base-year dollars. We need base-year dollar values because the expenditure theory applies to the actual value accomplished.

We present three expenditure models: Beta, Sech-Squared, and Rayleigh.

The Beta Model

Expenditures may be modeled with the Beta model.

$$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}t^{a-1} \bullet (1-t)^{b-1}$$

where Γ is the gamma function, $a \geq 0$, $b \geq 0$, and t = time (Jarvis, 1999). Jarvis and Pohl stated, "The beta curve provides great flexibility; however, a theoretical justification for using the beta curve is lacking" (Jarvis, 1999:9). Although the beta curve can fit the expenditure pattern associated with R&D program expenditures after completion of the program, we know of no method to estimate the model parameters a and b before program completion.

A useful model can forecast or project future needs. A "good" model offers the user a predictive tool in this application. Since the parameters of the beta model are currently estimated only after completion of the program, the beta curve provides little utility in projecting or forecasting program characteristics.

The Sech-Squared Model

Scaled expenditure may also be modeled using the Sech-Squared model. In equation form

$$f(t) = \frac{1}{4} \bullet \sec h^2\left(\frac{(at+c)}{2}\right)$$

where a and c are parameters selected to fit the expenditures.

Parr notes key characteristics of this model as being symmetric about its maximum and infinite tails (Parr, 1980:294). The model therefore has no specific starting date. According to Parr, this illustrates the fact that before programs officially starts, exploratory design studies and research projects aimed at specific aspects of the task are attempted (Parr, 1980:294). Unfortunately, these pre-start activities are not well documented nor have accounting systems been implemented to track these preliminary expenditures (Parr, 1980:295). Although the Sech-Squared Model accounts for these activities, little information actually exists to support building funding profiles from these pre-start activities.

In sum, the Beta Model provides little value as a predictive tool. The Sech-Squared Model does provide predictive capability. The difficulty of the Sech-Squared model comes with determining the parameters. For these reasons, our attention turns to the Rayleigh Model.

The Rayleigh Model

Norden noted that manpower utilization on software projects mirrored a life-cycle pattern that follows a distribution formulated by Lord Rayleigh (Putnam, 1978:347). Norden proposed using the Rayleigh Cumulative Distribution Function to model the manpower utilization during a project (Norden, 1970:122). Norden used the model to describe the quantitative behavior of various cycles of R&D projects (Norden, 1970:116,130). Because of Norden's efforts, the Rayleigh model is sometimes referred to as the Norden/Rayleigh model (Putnam, 1978:347).

Putnam applied the Rayleigh model to software system manpower (Putnam, 1978:346). He tested the Rayleigh model against the man-year budgetary data for about 50 systems of the Computer Systems Command and discovered that the project follows this life-cycle model remarkably well (Putnam, 1978:348).

Putnam was able to generate manpower, instantaneous costs, and cumulative costs of a software project at any time *t* by using the Rayleigh equation (Putnam, 1978:352). The ability to generate cumulative costs for a project is a key component of what this thesis effort hopes to achieve. The following paragraphs provide detail about how Putnam's achievement was adapted to defense programs.

Watkins applied the Rayleigh model to defense acquisition data and concluded the Rayleigh modeled the earned value of contracts well. Watkins applied the Rayleigh model to a Helicopter Engine contract and a Cruise Missile contract and found that differences in actual and modeled expenditures were not statistically significant (Watkins, 1982:79).

Lee, Hogue, and Gallagher (Lee, 1997:31) report Lee, Hogue, and Hoffman's unpublished conclusions that the Rayleigh function modeled outlays of a wide variety of defense acquisition contracts. Their research demonstrated that the Rayleigh Model fits contract expenditures expressed in base-year dollars of 20 defense R&D acquisition programs. This substantiates the claim that cumulative constant dollar R&D expenditures for defense programs may be modeled with the Rayleigh cumulative expenditure function (Gallagher, 1996:52).

Based on previous research (Putnam, Watkins, Lee, and Gallagher), we accept the Rayleigh model is useful as a predictive model for R&D expenditures for defense programs. The results of these studies reasonably affirm justification to use the Rayleigh model.

Before turning our attention to the Rayleigh equations, we summarize the Rayleigh research. Putnam was able to forecast costs for a software projects using the Rayleigh equation. Later, Watkins in one paper and Lee, Hogue, and Gallagher in a subsequent paper demonstrated that the Rayleigh function also models defense program expenditures. With this understanding the theoretical basis for this thesis effort—using the Rayleigh function to model funding curtailment—is now plausible. Our attention now turns to the function of the Rayleigh Model itself.

<u>The Weibull and Rayleigh Functions Are Related</u>. The Rayleigh model is a degenerative of the Weibull model. The Rayleigh Cumulative Distribution Function is

$$F(t) = 1 - e^{-at^b}. \qquad (3)$$

where a is the scale parameter and b is the constant of 2. If the b parameter is not a constant of 2, the equation is known as the Weibull Cumulative Distribution Function (CDF). We present and apply the Weibull distribution with the addition of a location parameter later.

<u>The Cumulative Distribution Function</u>. Figure 2-1 presents the Rayleigh cumulative distribution function using (3) with a=0.05 and b=2. This figure depicts how cumulative percent of expenditures for a specific R&D project are incurred.

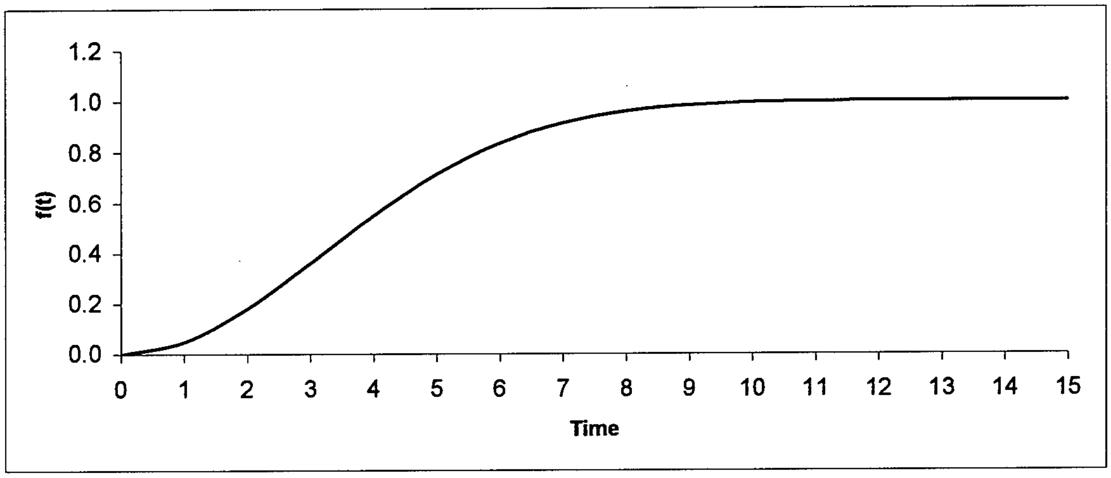

Figure 2-1. Rayleigh Cumulative Distribution Function

The Probability Distribution Function. The derivative of (3) provides the Rayleigh probability distribution function, shown in Figure 2-2 for the same parameters. The probability density function illustrates the rate at which percent of funds are expended.

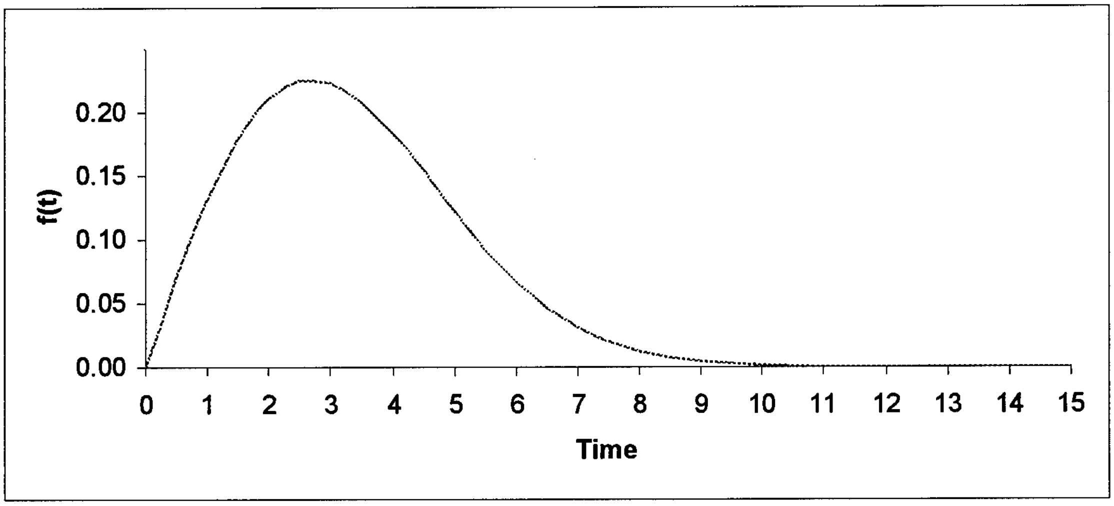

Figure 2-2. Rayleigh Probability Distribution Function

The Parameters of the Rayleigh Model. The cumulative distribution function graphed in Figure 2-1 uses the shaping parameter *a* to determine the curve. We use a second parameter to scale the Rayleigh PDF to program cost. Norden describes the *a* parameter as the coefficient that determines the month (time period) in which manpower utilization is greatest (Norden, 1970:126). The *a* parameter determines the steepness of the cumulative distribution function. Figure 2-3 demonstrates the increasing steepness when the *a* parameter is increased using (3).

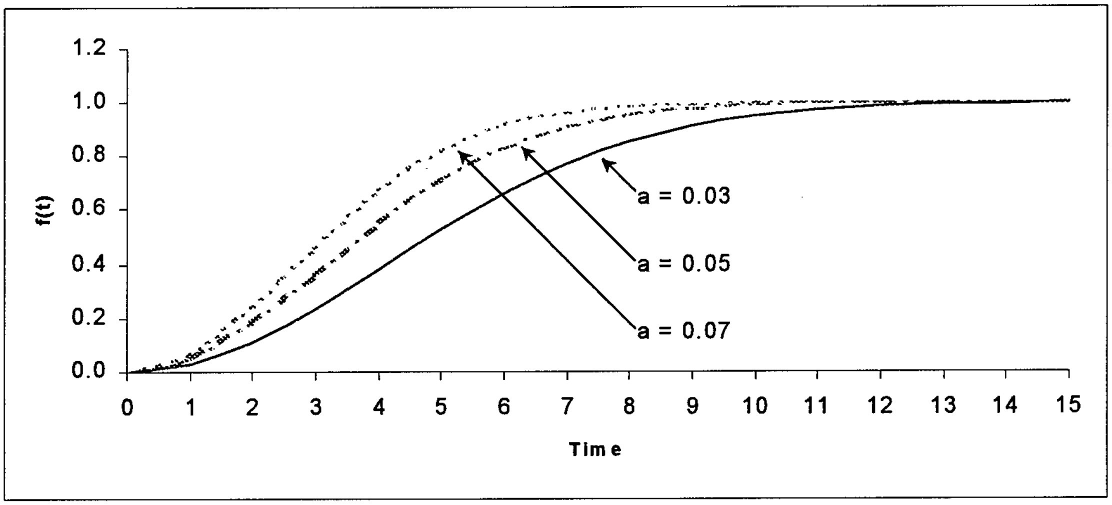

Figure 2-3. Effects of the *a* parameter on the Cumulative Distribution

The other parameter, *d*, is the cost scaling parameter. The results of (3) fall somewhere between 0 and 1. The *d* parameter scales this answer to the projected cost of the program. The complete Rayleigh cost model is

$$E(t) = d\left(1 - \left(e^{-at^2}\right)\right), \qquad (4)$$

where *d* is the cost of the program and the variable *t* represents the time from inception of the R&D effort to completion (Lee, 1997:31).

The Rayleigh expenditure rate for a modeled program is the derivative of the Rayleigh cost model (4),

$$f(t) = 2adte^{-at^2}. \tag{5}$$

Setting (5) equal to 0 and solving for *t* provides the apex point of the expenditure rate with respect to time. As the *a* parameter increases, the peak of the expenditure rate is earlier and higher in the program's timeline—as shown in Figure 2-4. For programs of the same value (in base-year dollars), if the rate of spending for a program is higher, the time for the program to reach maturity decreases. The shorter duration results because larger amounts of money are expended in the earlier phases of the R&D program.

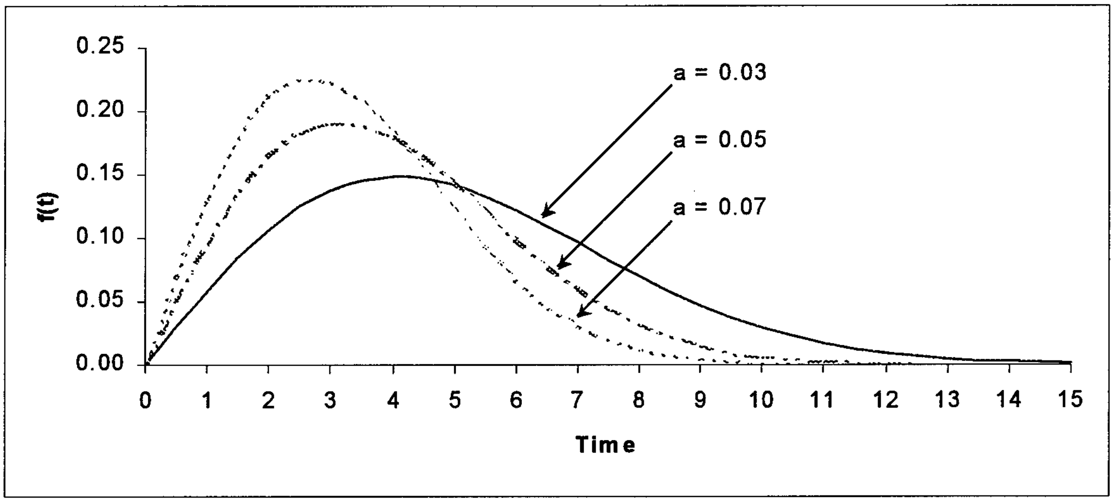

Figure 2-4. Effects of the *a* parameter on the Probability Distribution

The expenditure distribution provides key information about the modeled program. Based upon the shape of the expenditure distribution, we may determine the time of peak expenditure, the magnitude of the peak expenditure, and the projected program conclusion. By manipulating any one of these factors, holding *d* constant, one can determine the other factors. For example, if the peak expenditure is changed from year 3 to year 4, one can rework the expenditure distribution and determine a new projected project conclusion time and remaining expenditures assuming the overall program cost and requirements remain constant. A simple example will suffice.

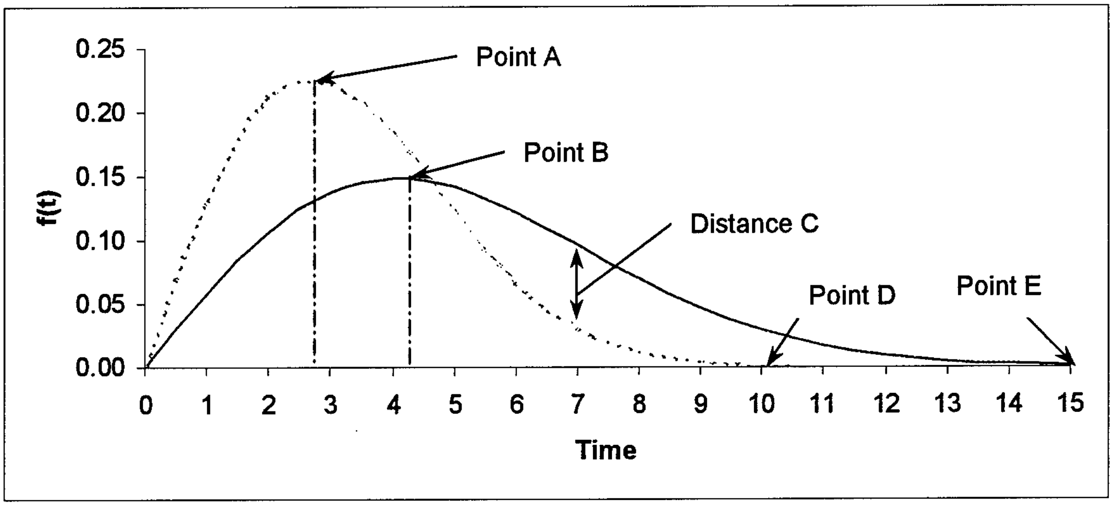

Figure 2-5. Hypothetical Modeled Program

Figure 2-5 presents a Rayleigh cost model for a hypothetical modeled program (The parameter *d* is set to one, so percent of expenditures are depicted). By changing the projected conclusion of the hypothetical R&D effort from Time 10 (Point D) to Time 15 (Point E) several consequences occur. First, the peak expenditure period changes from approximate Time 2.75 (Point A) to approximate Time 4.25 (Point B). Second, the amount at the peak expenditure time period is reduced. Finally, the expenditures at Time 7 increase from previous amounts (Distance C). The expenditures for all intervals along the time horizon need to be recalculated.

Determining the *a* Parameter. The shift in the Rayleigh probability density function signifies that the *a* parameter has changed. Two techniques exist for determining the appropriate a parameter: one based on the *time of peak expenditure rate method* and another based on the *expected completion time method.* Following Lee, Hogue, and Gallagher, we describe the characteristics of the Rayleigh model, explaining in detail the implications of changes in the cost model (Lee, 1997).

The Time of Peak Expenditure Method. This method determines the a parameter by establishing the time point at which peak expenditures of an R&D program occurs. Asserting the point of peak of expenditure can be estimated with some degree of reliability. For example, the peak expenditure rate in aircraft R&D programs comes approximately at the time of the first test flight (Lee, 1997:32). The time of peak rate expenditure, t_{peak}, may be related to the a parameter by setting the derivative of Equation (5) to zero. If the time of peak rate of expenditures is known, the appropriate shape parameter a is determined with Equation (5) (Lee, 1997:32).

$$a = 0.5t_{peak}^{-2}. \qquad (6)$$

The Expected Completion Time Method. The right side of the Rayleigh has an infinite tail. Therefore, a point must be determined that can be considered the end of the project. Lee, Hogue, and Gallagher define the time of final development, t_{final}, of a project in the Engineering, Manufacturing, and Development (EMD) phase when the Rayleigh cumulative distribution function shows 97% of the cumulative Research and Development (R&D) expenditures (expressed in base-year dollars) has been reached (Lee, 1997:32; Gallagher, 1996:52). In equation form

$$D = E(t_{final}) = 0.97d, \qquad (7)$$

where D is the total R&D program cost in constant dollars and d scales the Rayleigh cumulative distribution function to costs. Lee, Hogue, and Gallagher (Lee, 1997:33; Gallagher, 1996:52) demonstrate that the a parameter can be obtained using the formula

$$a \approx 3.5 t_{final}^{-2} \, . \qquad (8)$$

Therefore, given a projected completion time, the Rayleigh shape parameter may be determined with (8).

In summary, utilizing either the *time of peak expenditure method* or the *expected completion time method* determines an appropriate a parameter. The a parameter coupled with the total program cost (expressed in base-year dollars) provides enough information to use the Rayleigh cost models. Once the model parameters are estimated, cost analysts perform manipulations to forecast consequence of various "what-if" scenarios. Our discussion now turns to the characteristics of the Rayleigh Expenditure Distribution.

Characteristics of the Rayleigh Expenditure Distribution. The Rayleigh model's initial increase, peak, and trailing decrease are indicative of certain characteristics of the modeled program. At the beginning of a project, defined to be time 0, no effort or expenditures have been expended. Appropriately $E(0) = 0$, as defined in (4).

The period of increase in the Rayleigh expenditure rate distribution, the point beginning with inception and ending at the apex of the peak, is determined by how skill acquisition is accomplished. The Rayleigh model is developed as the product of two functions. The Weibull, and hence Rayleigh, cost models can be derived from the assumption that the rate at which work is completed is a function of performance and remaining work. Define the percent work remaining as $w(t)$ and performance as $p(t)$, both at time t. Then

$$\frac{dw(t)}{dt} = p(t)[1 - w(t)],$$

which may be solved for $w(t)$. Let $z(t) = 1 - w(t)$, so $\frac{dz(t)}{dt} = \frac{-dw(t)}{dt}$ and

$\frac{dz(t)}{dt} = -p(t)z(t)$. Integrating, we obtain $\ln(z(t)) = -\int_{\tau=0}^{\tau=t} p(\tau)d\tau$. We evaluate both sides to the power of the base of the natural logarithm, $z(t) = e^{-\int_{\tau=0}^{\tau=t} p(\tau)d\tau}$. We substitute back in percent of work to obtain

$$w(t) = 1 - e^{-\int_{\tau=0}^{\tau=t} p(\tau)d\tau}.$$

We define performance on the program for any given time as a constant multiplied by time to constant power, $p(t) = ct^b$. Since $\int_{\tau=0}^{\tau=t} p(\tau)d\tau = \int_{\tau=0}^{\tau=t} c\tau^b d\tau = \frac{c}{b+1}t^{b+1}$,

$w(t) = 1 - e^{-\frac{c}{b+1}t^{b+1}}$. With linear growth in performance over time, b=1 and a=c/2, we obtain the Rayleigh cumulative distribution function shown in (3). If performance improves with time to a power other than one, we have derived percent work complete according to a Weibull cumulative distribution function.

The Rayleigh model begins at zero and has an infinite right tail. At the initial stage, skill levels are zero. As time progresses, the amount of skills, represented as *2at*, available increase linearly. The *a* parameter governs the slope of the line as time increases. Parr states that the initially rising work rate is due to linear skills acquisition curve governing the "skill" available for solving problems (Parr, 1980:291). The level of skills available to the project grows as time on a particular project increases.

One of the assumptions of the Rayleigh model is there is a linear skill acquisition. Lee, Hogue, and Gallagher support this assumption (Lee, 1997:32). Resources, money in this case, are needed to fuel the skills acquisition process.

Similarly, as time progresses, the percent of work remaining decreases. As time progresses and skills to solve those problems increase, fewer and fewer problems remain. Parr sums this point up stating that the decay in the work rate at the conclusion of the project is due to the exhaustion of the problem space (Parr, 1980:291). Initially, the skills acquisition dominates and the rate of expenditures increases. Eventually, the percent of work remaining dominates and the rate of expenditures steadily decreases. The peak of the Rayleigh expenditure rate is where the dominance switches from skills acquisition to percent work remaining.

Unlike the left tail, the right tail continues infinitely. In a strict sense, this means that the R&D project never terminates. A never-ending project is simply not reality. For this reason a completion time has to be fixed. Lee, Hogue, and Gallagher (Lee, 1997:32) fix this point when 97% of the expenditures for an R&D project have been reached (7).

Figure 2-6 presents a graph of the terms $2at$ and e^{-at^2} separately and their product.

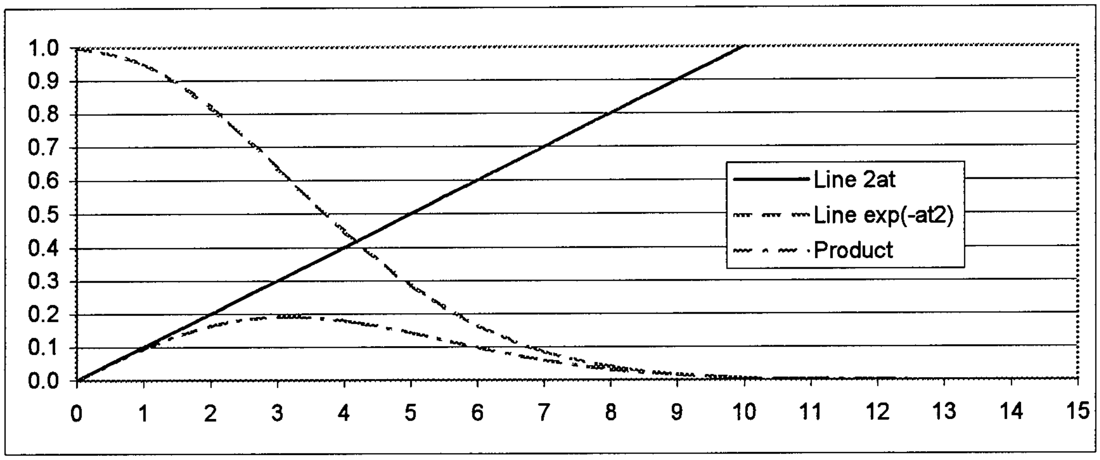

Figure 2-6. Components of the Rayleigh Function

Note the line *2at* continually increases as time progress. As stated before, the graph of e^{-at^2} decreases monotonically. The probability density function is the Product line in Figure 2-6.

Up to this point, the discussion has explored the Rayleigh function, the function traits, and the methods of determining its parameters. The Rayleigh Model uses these characteristics to aid in forecasting expenditures requirements for programs.

The Rayleigh function has the shape parameter set to a constant of 2. This makes the model somewhat rigid in its ability to model programs. The Rayleigh function forces a proportionate tail using the peak expenditure point as the start. In actuality there are programs where a proportionate tail is not derived from the point of peak expenditures. For example, a program may have a peak expenditure during one time period and a very short tail—program expenditures stop shortly thereafter. The Rayleigh function would not provide an accurate model of reality in this case because of its rigidity tied to the constant shape parameter. The resolution to this predicament is to allow the shape parameter to be variable, which takes us from the Rayleigh function to the Weibull function.

The Weibull Function

The form of the Rayleigh Equation shown in (3) is prevalent in the literature cited up to this point (Norden, 1970:122; Putnam, 1978:349; Lee, 1997:30; Gallagher, 1996:52). This equation assumes that program funding and manpower start with the inception of the program. Some programs begin with one or more years of, in essence, insignificant funding. Therefore, we model resulting expenditures with a location parameter, which disregards the starting time with insignificant funding.

In the Rayleigh functions, the shape parameter fixes the time of peak expenditures and program completion, as seen in (6) and (8). Since some programs, particularly those being restructured, do not follow this fixed pattern, we use a three-parameter Weibull distribution (Hines, 1980:165).

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}} \qquad (9)$$

where

$t = time$

$\gamma = location$

$\delta = scale$

$\beta = shape$

With the exception of the location parameter, (3) and (9) are equal. The shape parameter in (3), b, is equal to the shape parameter of (9), β. The scale parameter of (3),

a, is equal to $1/\delta^2$ of (9). The time parameter, t, of (3) is shown as t in (9). Equation (9) introduces the location parameter, γ, to the Weibull function. The location parameter allows the Weibull function to model program expenditures when programs are initiated but do not receive significant funding until a later year. The Weibull shape parameter allows skills acquisitions at other than linear rates. In addition, the Weibull time of peak expenditures does not fix the completion time of the program. The Weibull cost model is

$$E(t) = d\left(1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^{\beta}}\right). \qquad (10)$$

Using the Weibull function allows expenditure profiles to be modeled with a greater degree of accuracy than the Rayleigh function.

Relating Expenditures to Funding (Lee, 1997:34-38)

R&D programs in the Department of Defense receive budget authority or total obligation authority (TOA) that may be spent over several years. Since this funding provides the resources necessary to carry out the program over multiple years, the OSD comptroller publishes standard outlay (expenditure) patterns for these programs as part of the annual budget. The outlay pattern is the targeted fixed percentage of funds to be spent each year.

Modeled annual outlays (expenditures) are calculated in base-year dollars, defined as $\hat{O}_i$ for the ith year of the program. The hat indicates modeled values as opposed to those derived from the budget with (1) and (2). The difference between the current and previous modeled cumulative expenditures can be put in formula form as

$$\hat{O}_i = E(t_i) - E(t_{i-1}) \qquad (11)$$

where E is the cumulative expenditures in (10) and time t_i is the end of the fiscal year i.

The Rayleigh model provides the appropriate outlay profile needed for an R&D program. By knowing the parameters discussed previously, the model provides an outlay profile sufficient for the program to progress at a desirable rate. Lee, Hogue, and Gallagher provided an excellent example in their article that, at this point, needs to be understood. Their example with some slight modifications to aid understanding is as follows (Lee, 1997:34):

> As an example, consider a hypothetical program with EMD expenditures of a billion dollars (expressed in terms of millions of dollars) occurring over 10 years. The base year for the program is 1995. Since $t_{final} = 10.0$ and $D = 1{,}000$, the Rayleigh model parameters calculated with [(7)] and [(8)] are $d = 1030.93$ and $a = 0.035$. Table 2-1 shows the cumulative expenditures calculated with the Rayleigh model in [(4)] and these parameters. The annual outlays were calculated with [(9)]. We applied Navy raw inflation indices, shown in the last column, to covert to outlays in then-year dollars.

Table 2-1. Example Outlay Profile

Year	Time	Cumulative Rayleigh (Base-Year Dollars)	Annual Outlays (Base-Year Dollars)	Annual Outlays (Then-Year Dollars)	Inflation Index
1995	1	35.46	35.46	35.46	1.0000
1996	2	134.68	99.22	102.20	1.0300
1997	3	278.57	143.89	152.65	1.0609
1998	4	442.05	163.48	178.64	1.0927
1999	5	601.17	159.12	179.09	1.1255
2000	6	738.50	137.33	159.20	1.1593
2001	7	845.40	106.90	127.65	1.1941
2002	8	921.18	75.78	93.20	1.2299
2003	9	970.39	49.22	62.35	1.2668
2004	10	999.80	29.40	38.37	1.3048
Total			999.80	1,128.80	

From their example several important points are made. The Cumulative Rayleigh provides the funding needed for the program's success in base-year dollars

The annual outlays are presented in base-year and then-year dollars in the fourth and fifth columns, respectively. In other words, the sample program needs $143.89M in FY95 dollars or equivalently $152.65M in then-year dollars during the third year—program funding is given in then-year dollars. One would expect the program manager to obligate an amount equal to annual outlay in then-year dollars. This is the money the program manager would expend each year since these figures have been adjusted for inflation.

Up to this point, we have developed an expenditure profile. We know what the program spends each year. The task is how to convert the projected expenditures into a budget profile. What distinguishes an outlay (expenditure) profile from a budget profile?

Outlays are the monies flowing from a program. These monies come from different obligation authorities in different years. The outlay (expenditure) profile is based on an amalgamation of all these varied years of funding.

Our approach must aid in build the necessary total obligation authority (TOA) or budget profile. This profile is not a straight conversion of the outlay profile developed above. The entire appropriation for large-dollar projects cannot be spent during the fiscal year in which it was authorized. Certain amounts of the appropriation are allotted to the program each fiscal year.

For example, the OSD comptroller established outlay rates of 43.10, 44.55, 8.75, and 3.60 (expressed as percents) for the first through fourth year respectively in the FY 2000 National Defense Budget for RDT&E programs (Outlay Rates, FY2000). Although a program may have $10M appropriated to it in a year, the program manager is encouraged only to obligate (expend) $4.3M during the first year because the OSD comptroller established a 43.10% outlay for the first year. During the second year of the program, the program manager obligates about $4.45M from the first year budget authority. If the program manager needs $10M in total expenditures for the second year of the program, he should request $12.88M in appropriations for the second year. The calculation is as follows

$$\frac{(\$10\text{M needed - }\$4.45\text{M from first year appropriations})}{0.4310\text{ first year outlay rate}} = \$12.88\text{M}.$$

The type and complexity of the calculations described above increase in proportion to the length and size of the R&D program. From the simple example above, it can be noted that balancing the OSD comptroller outlay rates and the different years' authorizations become more difficult as the program continues on in years.

It should also be apparent the complex task inherent in developing a budget profile. The outlay pattern can be easily forecasted with a Rayleigh or Weibull model. The challenge comes in translating the outlay pattern into a budget profile taking into account various appropriation years tied to different outlay rates. The analyst knows what outcome is wanted (the expenditure profile) and must now develop a budget to match the expenditure profile. Lee, Hogue, and Gallagher describe an approach. We present this as an integral part of our approach.

Their method uses constrained nonlinear estimation to build the budget profile. In laymen's terms, this means that we change various inputs simultaneously that may impact other inputs until an optimal solution is reached. They substitute budget estimate $\hat{B}_i$ into (1) and (2). We select the budget estimate that minimizes the sum of squares error between the projected outlay profile based on the budget and the projected Weibull expenditure values. We select the revised budget value that

$$\min \sum_{i=1}^{N+J-1} \left(\tilde{O}_i - \hat{O}_i\right)^2 \qquad (12)$$

subject to $\hat{B}_i \geq 0$ and where $\hat{O}_i$ is the outlay (expenditure) profile needed as computed using the Weibull cost model in (9) and (10) and $\tilde{O}_i$ is the budget outlay for each *i*th year from (1) and (2). $N+J-1$ is the total program and outlay years we are trying to calculate.

Chapter Summary

In summary, the Rayleigh model aids in determining an expenditure profile for an R&D program. This assumption is based on several fundamental insights detailed in this chapter. First, Putnam derived cumulative costs of a software project using the Rayleigh function. Lee, Hogue, and Gallagher concluded the Rayleigh function modeled outlays of a wide variety of defense acquisition programs. Therefore, using the Rayleigh and its generalization, the Weibull function, is justified in modeling expenditure profiles.

This chapter also discusses the Weibull function and the flexibility it provides in modeling funding profiles. We explain how different parameters of the Weibull function are derived. We follow previous research efforts to define the end of a program as when 97% of modeled funds are expended.

We discuss expenditure profiles and budget profiles and the importance that outlay rates play evolving an expenditure profile from a budget profile. When program funding levels change, a new Rayleigh or Weibull model can be developed along with the associated funding profile.

In the next chapter we model the expenditure profile of a program before funding curtailment. The parameters derived in modeling the expenditure profile serve as the starting parameters for developing the parameters for the expenditure profile of the budget with the funding curtailment. The approach of Lee, Hogue, and Gallagher is used to determine a corresponding budget. Once this is accomplished, the original budget up to the time period of funding curtailment is combined with the budget derived from the model. This combined budget is in then-year dollars and provides the user with the cost consequences of the funding curtailment.

III. Research Methodology

Chapter Overview

Studies suggest the Rayleigh Model has predictive capability in large dollar procurements that are in the research and design (R&D) phase. This research develops a new R&D budget when funds for an on-going program are curtailed. Describing the methodology for this research is the focus of this chapter.

Program expenditures have been shown to follow the Rayleigh probability density function. When the program's funding profile changes, the associated Rayleigh probability density function changes. The task is to tie the original Rayleigh probability density function to the proposed Rayleigh density function taking into account that time and expenditure has already been incurred.

This chapter begins with an overview of the staggered funding profiles when funding curtailment is proposed. Afterward, this chapter describes the proposed methodology that provides the budget needed with considering funding curtailment. The pieces of information needed and the steps to apply the methodology are outlined. This chapter concludes with a brief summary of the methodology.

Curtailment Implications and Consequences

When funding cuts are directed or being considered, a new budget profile is calculated with different appropriations amounts being calculated with the outlay rates. The program manager may model the expenditures; the problem is determining the necessary funding by year with the current year's appropriations being reduced. The task is to create a model that allows these "budget drills" to be accomplished quickly and with relatively good accuracy.

A simple example provides an illustration of the complexities such a possible curtailment has in projecting future budgets. Figure 2-1 shows a cumulative expenditure profile of a program. We derived the parameters for this fictional program using (10) with d=100, $a = \frac{3.5}{10^2} = \frac{3.5}{100} = 0.035(7)$, and b=2. Now suppose the program modeled in Figure 2-1 is under consideration for funding curtailment in time period 6 as shown in Figure 3-1.

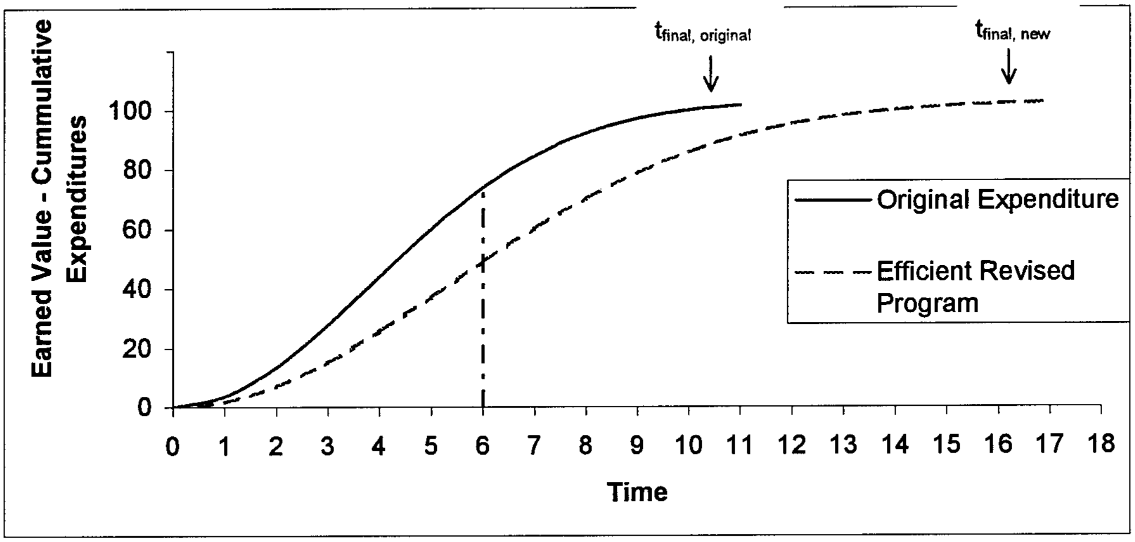

Figure 3-1. Profile of a Curtailed Program

Figure 3-1 shows a new dashed cumulative expenditure line with a new final time for the project due to the proposed funding curtailment. Had the curtailment been know from the project onset, the project would have had an expenditure profile of the dashed line instead of the solid line. The problem becomes complex because the program has actually spent money at the rate of the solid line (Original Expenditure) up to time period 6 and should now spend money at the rate indicated by the dashed line for all remaining time periods.

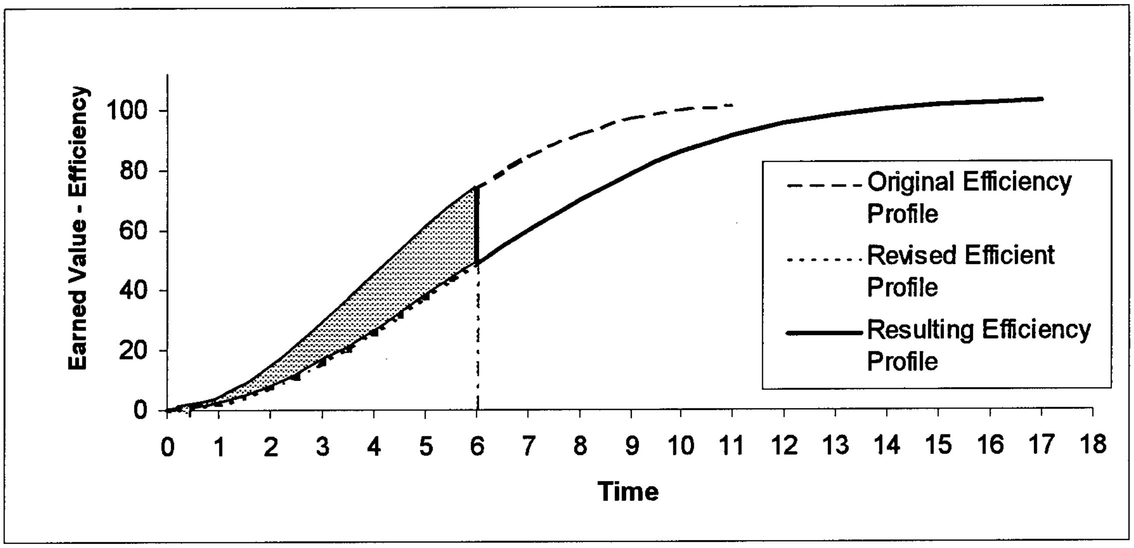

Figure 3-2. Staggered Efficiency Profile

Figure 3-2 presents the staggered efficiency profile. The program progresses along the solid, thick line until year six. At year six the curtailment occurs. The shaded triangle represents effort, which cannot be utilized by the program. There are insufficient funds available to take advantage of the work previously done and some earned value is lost. An example of such work is an engineering task that cannot be continued. The program does not receive the benefit of the task because the funding is curtailed. When the task is taken up again, effort is needed to reacquaint the engineers with the work previously done. We assume the task has to be accomplished because the requirement driving the task is still valid.

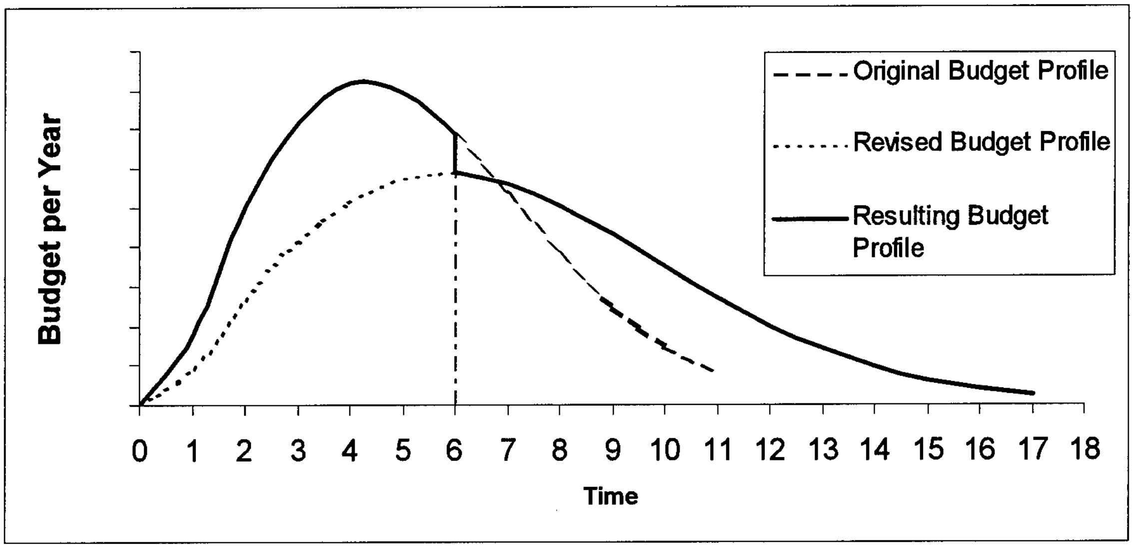

Figure 3-3. Staggered Budget Profile

Figure 3-3 presents the staggered budget profile. Both figures show the difficulty facing the program manager. The solid thick line represents the staggered budget profile the program now faces. The program has received a budget following the top curve up until time period 6. After time period 6, the remainder of the program follows the resulting budget curve.

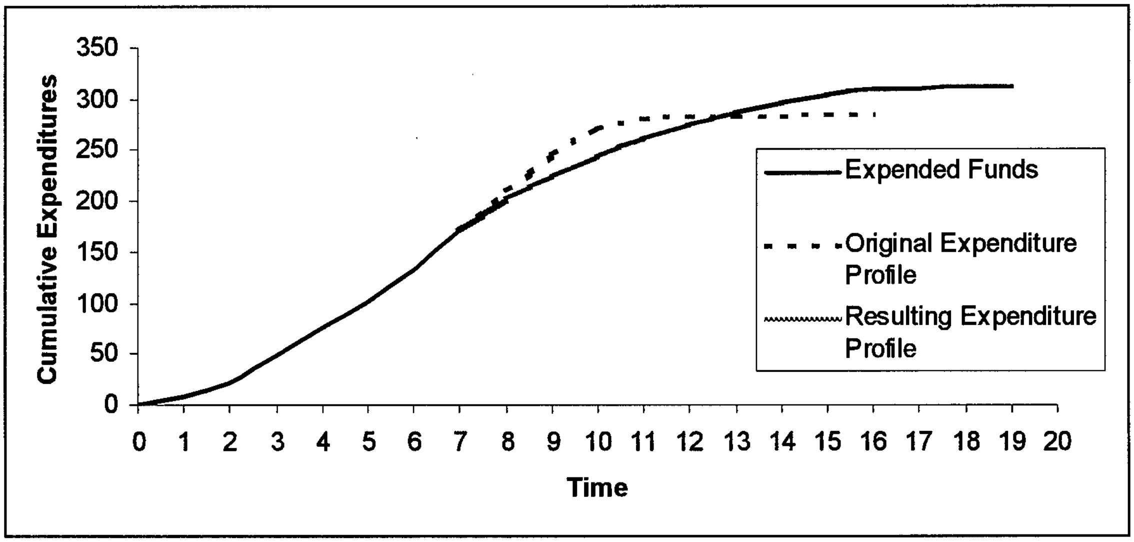

Figure 3-4. Expenditure Profile with Budget Curtailment

Figure 3-4 presents the cumulative expenditure profile for a sample program in base-year dollars with the funding curtailed in the seventh year. This figures shows the schedule growth from approximately year 16 to year 19. We also note the increase in expenditures for this program from approximately \$280M to \$310M base-year dollars.

The purpose of this thesis effort is to propose an approach that could quickly estimate the cost of the latter half of the staggered expenditure profile curve taking into consideration the expenditures already made by the program and the amount of the proposed curtailment.

In our proposed approach, the decrease at curtailment, year 6 in Figure 3-2, represents lost effort. This lost results from efforts interrupted and delayed. In Appendix A, we present an alternative approach that allows the expenditures to smoothly transition to the new outlay profile without any lost effort.

Information Needed

The proposed methodology requires three pieces of information: the original budget profile, the revised time to complete the R&D program, and the amount and year of the budget cut. The first piece of information needed is the current budget profile before the proposed curtailment. The program manager or program cost personnel keeps this information. They report it to congress in their annual SAR reports.

The second piece of information is to determine the revised final time, t_{final}^{rev}. We use Lee, Hogue, and Gallagher's definition of t_{final} given in (7). Applying the expenditure outlay rates to the current budget results in a by-year expenditure profile. We estimate the Weibull model parameters for this distribution. Based on these parameters and (7), we determine t_{final}^{rev}.

In practice the revised final time, t_{final}^{rev}, may be determined in two ways. The first and most convenient is that the user specifies the final time. In our tests, we specify the revised final time, t_{final}^{rev}, as the t_{final}^{org} for the original budget increased by the number of additional years of program R&D funding.

The other method involves spending caps. A program reviewer may determine the annual amounts of funding for a particular program are too high. Instead of focusing on the completion time of the project, the aim is to not exceed a certain expenditure amount during any program year. If this is the case, the experimenter selects a t_{final}^{rev} and applies our method. If the revised budget exceeds the ceiling amount for any year, the final time, t_{final}^{rev}, is extended. The same process is reiterated until the revised budget no longer exceeds the funding ceiling.

The final piece of information needed to forecast the budget profile when considering funding curtailment is the amount of the proposed curtailment. This figure is usually reported in budget figures and not in expenditure figures. For the proposed methodology, budget figures suffice.

The Methodology

With these three pieces of information (the original budget, revised final time, and funding curtailment), and analyst may apply our approach. The budget for a program is reported in then-year dollars. The first step in the methodology is to change the budget then-year profile into the expenditure base-year profile using (1) and (2).

The next step is to fit the Weibull model by estimating the five parameters in (10) for the original expenditure profile. We find the Cost parameter by taking the summed total of the base-year expenditures and dividing by 0.97 based on (7). The short reason is because when the Rayleigh or Weibull profile obtains a cumulative value of 97%, we define the R&D program to be complete. The Location, Shape, and Scale parameters are estimated based on the yearly expenditure data. We conduct a nonlinear search to minimize the squared error between the modeled and cumulative data to derive the Location, Shape, and Scale parameters. The original t_{final}^{org} is determined based on the modeled expenditures with (7) and (10) as

$$t_{final} = \delta\left(-\ln(0.03)\right)^{\frac{1}{\beta}} + \gamma . \tag{13}$$

The t_{final}^{rev} parameter is needed to derive the revised Weibull parameters and is calculated by adding the number of additional budget years to the original t_{final}^{org} for our tests.

Each time period of the program correlates to a certain expenditure. For example, the third year of a program may have \$12M in expenditures for that year alone and \$18M in cumulative expenditures from the first three years. The Weibull function also provides an expenditure amount for every time period, usually years, t_i. Subtracting cumulative expenditures of the year under consideration from the previous year gives the yearly expenditure amounts. Each time period of the program, usually given in years, has a cumulative expenditure amount correlated to it. By manipulating the Location, Shape, and Scale parameter, the amounts provided by the Weibull function change to be come closer and closer to the actual amounts. The method we proposed is to minimize the sum of the square difference between the actual expenditure amounts and expenditure amounts generated by the Weibull function. We select the Weibull parameters γ, δ, and β to

$$\min \sum_{i=1}^{N+J-1} \left(\widetilde{O}_i - \hat{O}_i\right)^2 \qquad (14)$$

where N is the number of budget years, J is the number of outlay years, and $\widetilde{O}_i^{org}$ is the expenditures for ith year in base-year dollars from (1) and (2) and $\hat{O}_i^{org}$ is the expenditure amount generated by the Weibull function in base-year dollars from (10) and (11).

We use Microsoft Excel's Solver feature as the primary utility for our methodology. Several other nonlinear search packages could also solve this problem. The target cell is the sum of all of the squared error. Solver is set to minimize this value. Solver manipulates the cells containing the Location, Shape, and Scale parameters to find the minimum value of the sum of squared errors.

The initial values for the search are important. If the initial values are too far off from the global optimal values, solver may stop searching at a local optimum. We used zero for the initial value for the Location parameter. This assumes the program starts in its first year of funding. Several programs had a few years of little funding after which they did receive significant funding. We use 2 as the initial value for the Shape parameter. Chapter Two discusses that the Rayleigh model provides for efficient funding of a program, and the Rayleigh model is a Weibull function with a Shape parameter of 2. By starting the search for a Shape parameter at 2, the assumption is the program funding supports Rayleigh expenditures. The initial value of the Scale parameter should be set so as to extend the duration of the Weibull model to the duration of the actual program. The estimation step produces the Location, Shape, and Scale parameter for the original expenditure profile. The final time of the original program is calculated with (13).

The next step is incorporating the proposed budget cut and the revised program duration. We substitute the proposed budget in the appropriate year into (1). A new expenditure profile is created using this proposed budget up to the year of curtailment—hereafter referred to as the revised expenditure profile. For example, if the budget in year 6 of program is under consideration for curtailment, we build a partial expenditure profile using the original budget values for years 1 through 5 and the proposed budget for year 6.

The partial revised expenditure profile should be equal in dollar amounts for each year before the curtailment year. Because the outlay rate and inflation affect on the budget, we do not find a dollar for dollar reduction difference in the expenditure profile.

The next step is to fit the Weibull Shape and Scale parameters to this revised expenditure profile while retaining the Location and Cost parameters at their original values.

The t_{final}^{rev} parameter for the revised budget is the original t_{final} parameter plus the increase in program duration. The unit of time should be consistent. If the original t_{final} parameter is given in years and the revised t_{final}^{rev} parameter is given in months, the revised t_{final}^{rev} parameter will need to be converted to years.

We use a similar method of fitting the Weibull Shape and Scale parameters as determining the original Weibull parameters. Instead of minimizing the sum of all the squared errors as was previously done, the objective is to minimize the squared error for the year of the proposed budget curtailment. Also the actual budget expenditures must exceed the Weibull forecasts for any year prior to the proposed curtailment.

We estimate revised Weibull shape and scale parameters with

$$\min\left(\tilde{O}_i^{rev} - \hat{O}_i^{rev}\right)^2 \qquad (15)$$

subject to

$$\tilde{O}_i^{rev} \le \hat{O}_j^{org} \qquad \text{for } j = 1, \ldots, I$$

$$t_{final}^{rev} = t_{final} \ \text{ from (13)}$$

We use the original location, γ, and cost scale, d, parameters

The initial values for the Shape and Scale parameters are the Shape and Scale parameters from the original budget. Since the original and revised profiles support the same program, we expect the parameter values are close to the original parameters. This estimation obtains the Shape and Scale parameters of the revised expenditure profile.

The next step is to use the revised parameters along with the Location, Cost, and t_{final}^{rev} parameters and build an expenditure profile. The Weibull function provides a cumulative expenditure profile. By subtracting previous time period cumulatives from each time period, the expenditures for just one time period can be obtained with (11). The resulting modeled expenditure profile is in base-year dollars.

The next step is to build a budget from this profile. In Chapter Two the complexities of taking an expenditure (outlay) profile and creating a budget profile are discussed.

Up to the year when funding curtailment is possible, the budgeted amount for each year has already been authorized and possibility spent. The objective is to build a budget taking into account what has already actually been appropriated with what effect of the curtailment. At this point we know what the budget was up to the year under consideration, we know what the proposed budget will be for the year in consideration, what we need is the budget for the year following the curtailment.

We solve for the budget in the out years that follows the revised Weibull cost model with (12). The revised budget profile incorporates the original budget from up to the year considered for curtailment, the curtailed budget, and the revised budget estimates for the remaining years. We create a revised budget as

$$\left\{B_1,\ldots,B_{i-1};B_i^{curtail};\hat{B}_{i+1},\ldots,\hat{B}_N\right\}. \qquad (16)$$

The result of this step is a budget that incorporates the money already spent, the curtailment of funds, and the remaining budget necessary after the proposed curtailment.

In summary, the methodology is:

- Change budget profile into an expenditure profile using outlay rates with (1)
- Change expenditure profile into base-year dollar by dividing each year by the appropriate inflation factor with (2)
- Derive Weibull parameters from the original expenditure profile
    - Cost parameter d as total program cost in base-year dollars divided by 0.97 based on (7)
    - Least squares estimation for Shape, Scale, and Location parameters with (14)
    - Final time, t_{final}^{org}, based on the estimated parameters with (13)

- Incorporate the proposed budget curtailment and determine new outlay in current year with (1)
- Specify revised completion time, we add the number of additional budget years to the original t_{final}^{rev}
- Estimate Weibull Shape and Scale parameters subject to revised expenditure less than original expenditure and meeting revised completion time with (15)
- Develop a new budget for revised expenditures with (12)
- Build a revised budget using money already spent for past years, proposed curtailment for the planned year, and the new budget for future years as shown in (16)

Chapter Summary

This chapter describes the proposed methodology that determines a revised budget for curtailment of an ongoing R&D program. This chapter contains an explanation of each step along with a short summary of the methodology.

In Chapter Two we discuss the theory behind Rayleigh and Weibull function as it pertains to defense acquisition. In the following chapter, Chapter Four, we provide the results of applying this methodology to actual programs.

IV. Results and Discussion

Chapter Overview

Chapter Three outlines our methodology to forecast the budget needed when considering funding curtailment. This chapter explains the program selection criteria to test this methodology and presents the different programs used to validate the methodology. The definition of research success is also discussed. For each program, we provide the amount of the curtailment and the schedule slip occurring at the time of the curtailment. The Weibull parameters used in modeling the programs are given for both the original budget profile and the profile incorporating the curtailment (labeled 'revised'). This chapter concludes with synopsized results from applying the methodology to historical completed programs.

Program Criteria

This section lists and provides rationale for programs that are candidates to validate the proposed methodology.

The first criterion is the initial program Selected Acquisition Report (SAR) is after the early 1980s. The Selected Acquisition Report is submitted to congress on certain large dollar programs. Before the early 1980s, the format of the SAR was not standardized. After the early 1980s, the budget data was reported in a tabular format that allows comparison from year to year. We select only programs that adopted the standardized format.

The second criterion is the program completed RDT&E funding before 1999. This criteria allows us to know how the final RDT&E funding concluded for the program—which provides a baseline that we can compare all previous reports. We considered comparing against the next year Selected Acquisition Report, but we found the budget a year after a curtailment often constituted an underfunded the program.

The third criterion is funding curtailment occurred. Funding must be delayed from at least one year to later in the program. We found these programs by comparing the annual RDT&E tables with the final SAR.

A fundamental, critical assumption is that scope/requirements for the program have not changed. A requirements change limits the ability to assess any funding increase directly attributable to delaying money from one year to latter years. We eliminate programs that had significant decrease in base-year dollar totals since this indicates a reduction in program requirements. In addition, if the SAR indicates a change in requirements, we eliminate the program.

These criteria provide the guidelines in selecting programs to test the methodology. We treat the sum of each department's portion of joint programs as a single program. This prevents examining inter-service transitions of funding for joint programs. We applied the lead services outlay rates, shown in Table 4-1. Each row is the outlay rate over the fiscal years. Inflation factors are contained in Appendix B for these programs.

Table 4-1. Service Outlay Rates

Air Force

FY2001	FY2000	FY1999	FY1998	FY1997	FY1996	FY1995	FY1994	FY1993	Average	StDev
59.5	58.8	59.1	50.7	45.8	46.3	46.5	46.5	50.8	51.56	5.98
33.7	34.5	33.1	37.4	39.9	39.1	38.8	38.8	34.5	36.64	2.67
3.6	3.6	5.3	6.8	8.9	8.9	8.8	8.9	9.5	7.14	2.40
1.0	1.0	1.4	3.0	3.6	3.6	3.6	3.6	3.4	2.69	1.19
0.3	0.3	0.4	0.8	1.1	1.1	1.1	1.1	1.1	0.81	0.37
	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.35	0.05
		0.2		0.1		0.2		0.2	0.18	0.05
98.1	98.5	99.8	99.0	99.7	99.4	99.4	99.3	99.9	**99.37**	

Army

FY2001	FY2000	FY1999	FY1998	FY1997	FY1996	FY1995	FY1994	FY1993	Average	StDev
57.5	56.8	58.0	58.0	58.0	57.0	57.0	55.0	55.0	56.92	1.19
32.5	33.7	33.0	33.0	33.0	34.0	34.0	34.0	34.0	33.47	0.59
6.3	5.0	5.3	5.3	5.3	5.3	5.3	7.3	7.3	5.82	0.91
2.1	2.1	2.1	1.8	1.8	1.8	1.8	1.8	1.8	1.90	0.15
0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.80	0.00
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.50	0.00
		0.2		0.2		0.2		0.2	0.20	0.00
99.2	98.9	99.9	99.4	99.6	99.4	99.6	99.4	99.6	**99.61**	

Navy

FY2001	FY2000	FY1999	FY1998	FY1997	FY1996	FY1995	FY1994	FY1993	Average	StDev
59.5	59.3	60.5	58.0	55.9	55.9	54.0	55.0	55.0	57.01	2.35
31.4	33.6	32.5	33.1	31.5	31.5	32.4	33.4	33.4	32.53	0.89
5.9	4.5	4.5	5.4	8.2	8.2	8.0	7.8	7.8	6.70	1.61
1.9	1.0	1.0	2.0	2.0	2.0	2.0	1.3	1.3	1.61	0.45
0.7	0.3	0.3	0.6	1.1	1.1	1.1	1.1	1.1	0.82	0.35
	0.2	0.2	0.2	0.4	0.4	0.4	0.4	0.4	0.33	0.10
				0.6		0.2		0.2	0.33	0.23
99.4	98.9	99.0	99.3	99.7	99.1	98.1	99.0	99.2	**99.34**	

We used the average outlay rate shown in the second column from the right as the outlay rates for our programs. There was not a significant difference in the outlay rates from year to year. The first number in the average column is for the first year, the second number is for the second year, and so on. The averages do not sum to 100% percent. A small percent of money expires without being obligated.

Each programs inflation index is contained in Appendix B.

Success Defined

The purpose of the methodology is forecast a revised R&D budget. The forecasted budget is compared to the final RDT&E budget the deviation is reported. We define methodology success minimal deviation from the final reported RDT&E budget. In formula form

$$\frac{\left(\sum B_{Model} - \sum B_{Final}\right)}{\sum B_{Final}} \qquad (17)$$

where B_{Model} is the then-year dollar revised (Weibull) budget and B_{Final} is then-year budget of the final R&D Selected Acquisition Report. Method success is also calculated and reported in the chapter in base-year dollars.

For example, if the model forecasts \$115M as the budget in then-year dollars and the final RDT&E budget in the SAR contains \$120M in then-year dollars, then the model forecasted budget is –4.17% below (underestimated) the actual final RDT&E budget SAR. A reporting of 0% would mean the forecasted amount was exactly equal to the actual final RDT&E budget SAR amount—this would be a perfect forecast, which is the goal.

Programs

We selected programs based on the criteria outlined above. We considered 37 programs that had their initial budgets in standard format from the 1999 SAR database. Fourteen of these programs report a complete RDT&E phase before the 1999 SAR. Of these programs, we found five programs that show funding curtailment in one year and a return of funding in later years. Since one programs had repeated curtailments, we found six data points.

The MLRS-TGW Program. The MLRS-TGW program is an Army Missile program. The first SAR report for this program was 1984 with the final SAR for the RDT&E budget submitted in the 1991 SAR.

Table 4-2. MLRS-TGW Program 1985 SAR Then-Year Budget

Budget Year	1985 Budget	Final SAR Budget	Difference	Revised Budget
1980	0.5	0.5		0.5
1981	0.3	0.3		0.3
1982	1.0	1.0		1.0
1983	2.4	2.3	-0.1	2.4
1984	15.5	15.5		15.5
1985	24.5	24.1	-0.4	24.5
1986	30.2	27.1	-3.1	30.2
1987	42.7	39.3	-3.4	42.7
1988	52.2	23.6	-28.6	23.6
1989	47.6	39.1	-8.5	58.8
1990	52.0	41.7	-10.3	38.3
1991	29.0	41.7	12.7	32.4
1992		46.5	46.5	28.6
TOTAL	297.9	302.7	4.8	298.9

Table 4-3. MLRS-TGW Program 1985 SAR Base-Year Budget

Budget Year	1985 Budget	Final SAR Budget	Difference	Revised Budget
1980	0.7	0.7		0.7
1981	0.4	0.4		0.4
1982	1.1	1.1		1.1
1983	2.5	2.4	-0.1	2.5
1984	15.7	15.7		15.7
1985	24.0	23.6	-0.4	24.0
1986	28.9	25.9	-3.0	28.9
1987	39.1	36.0	-3.1	39.1
1988	46.5	21.0	-25.5	21.0
1989	40.5	33.3	-7.2	50.0
1990	42.9	34.4	-8.5	31.6
1991	23.1	33.2	10.1	25.8
1992		35.9	35.9	22.1
TOTAL	265.4	263.6	-1.8	263.0

Table 4-2 and 4-3 present the 1985 SAR budget in then-year and base-year figures respectively. The base year budget is derived using (2) and the inflation factors in Table B-7. It shows funding curtailment compared with the final SAR. In years starting with 1988 through 1990 there is a total curtailment of $47.4M (then-year dollars). The 1985 SAR shows $59.2M is added back into the program in the years 1991 and 1992 (then-year dollars). The program experienced a one-year schedule slip.

Table 4-4. MLRS-TGW Program Weibull Parameters

Program	Original					Revised				
	Year	Location	Shape	Scale	t_final	**Year**	Location	Shape	Scale	t_final
MLRS-TGW	1985	2.027306	3.24656	7.799445	13.50612	1985	2.027306	3.103132	8.328803	14.50612

Table 4-4 above presents the Weibull parameters for the 1985 SAR budget before the curtailment (Original) and when the curtailment is considered (Revised). The *location* parameter remains the same because the beginning of the program cannot change. The change in the *shape* parameter indicates the peak expenditure for this budget profile is now proportionally earlier than originally programmed but the program is longer. The *scale* parameter growth is indicative of the increased program duration. We increase the t_{final}^{rev} parameter by 1 year for the additional budget year.

The method forecasts 1.25% in then-year dollars and 0.23% in base-year dollars under the actual final budget for the MLRS-TGW program. For this program, the proposed method estimates a slightly larger (optimistic) budget.

<u>The ADCAP Program</u>. The ADCAP program is a Navy Munitions program. The first SAR report for this program was 1985 with the final SAR for the RDT&E budget submitted in the 1994 SAR.

Table 4-5. ADCAP Program 1986 SAR Then-Year Budget

Budget Year	1986 Budget	Final SAR Budget	Difference	Revised Budget
1979	17.9	17.9		17.9
1980	52.6	52.6		52.6
1981	90.6	90.6		90.6
1982	154.4	154.4		154.4
1983	180.4	180.4		180.4
1984	173.2	172.9	-0.3	173.2
1985	125.9	125.5	-0.4	125.9
1986	60.7	60.3	-0.4	60.7
1987	58.0	56.5	-1.5	58.0
1988	32.2	20.2	-12.0	20.2
1989	30.3	26.2	-4.1	34.0
1990	7.5	34.0	26.5	21.2
1991	29.9	56.7	26.8	20.6
1992	42.5	14.7	-27.8	17.7
1993	0.0	28.0	28.0	11.9
1994	0.0	26.7	26.7	19.8
TOTAL	1056.1	1117.6	61.5	1059.0

Table 4-6. ADCAP Program 1986 SAR Base-Year Budget

Budget Year	1986 Budget	Final SAR Budget	Difference	Revised Budget
1979	26.0	26.0		26.0
1980	68.6	68.6		68.6
1981	106.7	106.7		106.7
1982	170.1	170.1		170.1
1983	189.9	189.9		189.9
1984	175.5	175.2	-0.3	175.5
1985	123.4	123.0	-0.4	123.4
1986	58.1	57.7	-0.4	58.1
1987	53.1	51.7	-1.4	53.1
1988	28.7	18.0	-10.7	18.0
1989	25.8	22.3	-3.5	28.9
1990	6.2	28.1	21.9	17.5
1991	23.8	45.1	21.3	16.3
1992	32.8	11.4	-21.5	13.7
1993	0.0	21.2	21.2	9.0
1994	0.0	19.9	19.9	14.7
TOTAL	1088.8	1134.9	46.1	1089.6

Table 4-5 and 4-6 present the 1986 ADCAP program SAR budget in then-year and base-year figures respectively. Comparing the 1986 SAR to the final SAR indicates funding is delayed from the program and then returned in the later years. A reduction of $12M in then-year dollars occurs in 1988. The method underestimates the final budget for this occurrence by 5.24% in then-year dollars and 3.99% in base-year dollars. The ADCAP program schedule slipped by two years.

Table 4-7. ADCAP Program Weibull Parameters

Program	Original					Revised				
	Year	Location	Shape	Scale	t_final	**Year**	Location	Shape	Scale	t_final
ADCAP	1986	4.816773	0.47556	1.181748	21.34731	1986	4.816773	0.519409	1.655189	23.34731

Table 4-7 above presents the Weibull parameters for the ADCAP program budget before the curtailment (Original) and when the curtailment is considered (Revised). The *location* parameter remains the same because the beginning of the program has not changed. The change in the *shape* parameter indicates the peak expenditure for this program occurs later than originally programmed. The *scale* parameter growth is indicative of the overall program duration. The t_{final}^{rev} parameter increases by the noted schedule growth. The model for the ADCAP program gives an optimistic estimate.

The ADDS Program. The ADDS program is an Army Electronic program. The first SAR report for this program was 1983 with the final SAR for the RDT&E budget submitted in the 1994 SAR.

Table 4-8. ADDS Program 1986 SAR Then-Year Budget

Budget Year	1986 Budget	Final SAR Budget	Difference	Revised Budget
1981	15.8	15.8		15.8
1982	17.3	17.3		17.3
1983	34.1	34.1		34.1
1984	22.9	22.9		22.9
1985	23.9	23.9		23.9
1986	33.4	36.0	2.6	33.4
1987	35.7	38.0	2.3	35.7
1988	33.9	21.7	-12.2	21.7
1989	25.5	10.5	-15.0	15.5
1990	13.0	3.5	-9.5	17.1
1991	0.0	3.1	3.1	13.9
1992	0.0	6.1	6.1	12.1
1993	0.0	7.6	7.6	10.4
1994	0.0	14.7	14.7	7.9
1995	0.0	2.3	2.3	7.9
1996	0.0	1.9	1.9	3.0
TOTAL	255.5	259.4	3.9	292.6

Table 4-9. ADDS Program 1986 SAR Base-Year Budget

Budget Year	1986 Budget	Final SAR Budget	Difference	Revised Budget
1981	18.6	18.6		18.6
1982	19.1	19.1		19.1
1983	35.9	35.9		35.9
1984	23.2	23.2		23.2
1985	23.4	23.4		23.4
1986	32.0	34.5	2.5	32.0
1987	32.7	34.8	2.1	32.7
1988	30.2	19.3	-10.9	19.3
1989	21.7	8.9	-12.8	13.2
1990	10.7	2.9	-7.8	14.1
1991	0.0	2.5	2.5	11.0
1992	0.0	4.7	4.7	9.4
1993	0.0	5.8	5.8	7.8
1994	0.0	10.9	10.9	5.9
1995	0.0	1.7	1.7	5.7
1996	0.0	1.4	1.4	2.1
TOTAL	247.5	247.5	0.0	273.6

Table 4-8 and 4-9 present the 1986 SAR report budget profile in then-year and base-year dollars respectively and shows that funding curtailment occurs in 1988 through 1990. The 1986 SAR indicates that $12.2M in then-year dollars is removed from the program in 1988. The 1986 SAR shows $35.7M in then-year dollars returning to the program starting in 1991. This program experienced a schedule slip of 6 years. The method overestimates the final RDT&E budget by 12.80% in then-year dollars and 10.52% in base-year dollars.

Table 4-10. ADDS Program Weibull Parameters

Program	Original					Revised				
	Year	Location	Shape	Scale	t_final	**Year**	Location	Shape	Scale	t_final
ADDS	1986	5.13E-09	1.996983	6.5709	12.31621	1986	5.13E-09	1.536882	8.096528	18.31621

Table 4-10 presents the Weibull parameters for the ADDS program budget before the curtailment (Original) and when the curtailment is considered (Revised). The *location* parameter, in this case nearly 0, indicates the program receives significant funding immediately. The change in the *shape* parameter indicates the peak expenditure for the revised budget profile is proportionally earlier than originally programmed, but the program is longer. The *scale* parameter growth is indicative of the overall program duration. The t_{final}^{rev} parameter increases by the noted schedule growth—6 years.

The TRI-TAC Program. The TRI-TAC program is an Air Force Electronic program. The first SAR report for this program was 1983 with the final SAR for the RDT&E budget submitted in the 1989 SAR.Table 4-11.

Table 4-11. TRI-TAC Program 1983 SAR Then-Year Budget

Budget Year	1983 Budget	Final SAR Budget	Difference	Revised Budget
1973	1.8	1.8		1.8
1974	4.0	4.0		4.0
1975	7.4	7.4		7.4
1976	3.9	3.9		3.9
1977	5.0	5.0		5.0
1978	7.0	7.0		7.0
1979	4.4	4.4		4.4
1980	9.2	9.2		9.2
1981	7.4	7.4		7.4
1982	10.9	10.9		10.9
1983	25.4	22.8	-2.6	22.8
1984	7.9	7.6	-0.3	0.0
1985	4.1	3.7	-0.4	10.4
1986	3.0	1.0	-2.0	4.4
1987	4.4	1.0	-3.4	7.1
1988	4.7	2.7	-2.0	4.4
1989	4.8	1.2	-3.6	5.0
1990	0.0	3.5	3.5	3.4
1991	0.0	4.8	4.8	3.7
1992	0.0	5.0	5.0	2.0
1993	0.0	5.2	5.2	3.2
TOTAL	115.3	119.5	4.2	127.6

Table 4-8 presents the 1983 SAR then-year budget figures for the TRI-TAC program. The following table presents the budget figures for this program in base-year dollars.

Table 4-12. TRI-TAC Program 1983 SAR Base-Year Budget

Budget Year	1983 Budget	Final SAR Budget	Difference	Revised Budget
1973	4.2	4.2		4.2
1974	8.6	8.6		8.6
1975	14.4	14.4		14.4
1976	7.0	7.0		7.0
1977	8.5	8.5		8.5
1978	11.0	11.0		11.0
1979	6.4	6.4		6.4
1980	12.0	12.0		12.0
1981	8.7	8.7		8.7
1982	12.0	12.0		12.0
1983	26.7	24.0	-2.7	24.0
1984	8.0	7.7	-0.3	0.0
1985	4.0	3.6	-0.4	10.2
1986	2.9	1.0	-1.9	4.3
1987	4.0	0.9	-3.1	6.5
1988	4.2	2.4	-1.8	3.9
1989	4.1	1.0	-3.1	4.3
1990	0.0	2.9	2.9	2.8
1991	0.0	3.8	3.8	2.9
1992	0.0	3.9	3.9	1.6
1993	0.0	3.9	3.9	2.4
TOTAL	146.6	147.8	1.2	155.6

Table 4-12 presents the 1983 SAR for the TRI-TAC program in base-year dollars. Both tables show funding curtailment. Funding is reduced is in 1983. During this year, the program experiences four years of schedule slip. The method overestimates the final RDT&E budget by 6.74% in then-year dollars and 5.28% in base year (constant) dollars. For this program, the proposed method estimates a larger (conservative) budget.

Table 4-13. TRI-TAC Program Weibull Parameters

Program	Original					Revised				
	Year	Location	Shape	Scale	t_final	**Year**	Location	Shape	Scale	t_final
TRI-TAC	1983	1.145165	2.234193	10.79748	20.07753	1983	1.145165	1.982229	12.17772	24.07753

Table 4-13 above presents the Weibull parameters for the TRI-TAC program budget before the curtailment (Original) and when the curtailment is considered (Revised). The *location* parameter remains the same because the beginning of the program has not changed. The change in the *shape* parameter indicates the peak expenditure for this budget profile is proportionally earlier than originally programmed, but the program is much longer. As with previous modeled programs, the *scale* parameter increase indicates schedule growth of the program. The t_{final}^{rev} parameter increases by the noted four-year schedule growth.

The WAM Program. The WAM Program is an Air Force Electronic program. This program receives funding from all military services plus other defense agencies. The first SAR report for this program was in 1983 with the final SAR for the RDT&E budget submitted in the 1991 SAR.

This program shows funding curtailment in three years: 1983, 1989, and 1990. WAM program requirements were not finalized until 1985. Since we intend to model program with constant program requirements, we exclude the funding curtailment noted in 1983.

Table 4-14. WAM Program 1989 SAR Then-Year Budget

Budget Year	1989 Budget	Final SAR Budget	Difference	Revised Budget
1982	14.0	14.0		14.0
1983	17.8	17.8		17.8
1984	46.5	46.5		46.5
1985	76.0	76.0		76.0
1986	104.5	104.5		104.5
1987	138.4	138.4		138.4
1988	59.6	59.1	-0.5	59.6
1989	96.4	96.3	-0.1	96.4
1990	81.8	75.0	-6.8	81.8
1991	83.3	73.6	-9.7	83.3
1992	96.6	59.1	-37.5	59.1
1993	79.5	44.1	-35.4	74.9
1994	70.5	51.3	-19.2	52.8
1995	0.0	49.8	49.8	43.1
1996	0.0	44.4	44.4	45.7
1997	0.0	29.6	29.6	10.3
TOTAL	964.9	979.5	14.6	1004.2

Table 4-15. WAM Program 1989 Base-Year Budget

Budget Year	1989 Budget	Final SAR Budget	Difference	Revised Budget
1982	15.4	15.4		15.4
1983	18.7	18.7		18.7
1984	47.1	47.1		47.1
1985	74.5	74.5		74.5
1986	100.0	100.0		100.0
1987	126.7	126.7		126.7
1988	53.1	52.7	-0.4	53.1
1989	82.0	81.9	-0.1	82.0
1990	67.5	61.9	-5.6	67.5
1991	66.3	58.5	-7.7	66.3
1992	74.7	45.7	-29.0	45.7
1993	60.2	33.4	-26.8	56.7
1994	52.5	38.2	-14.3	39.3
1995	0.0	36.4	36.4	31.4
1996	0.0	31.9	31.9	32.8
1997	0.0	21.0	21.0	7.3
TOTAL	838.7	844.0	5.3	864.7

Table 4-14 and 4-15 present the 1989 SAR and shows $92.1M in then-year dollars being curtailed from the program starting in 1992 and $123.8M in then-year dollars reprogrammed into the program's later years. The WAM program experiences three years of schedule slip in the 1989 SAR. The method overestimates the final RDT&E budget by 2.53% in then-year dollars and 2.45% in base-year dollars.

Table 4-16. WAM Program 1990 SAR Then-Year Budget

Budget Year	1990 Budget	Final SAR Budget	Difference	Revised Budget
1982	14.0	14.0		14.0
1983	17.8	17.8		17.8
1984	46.5	46.5		46.5
1985	76.0	76.0		76.0
1986	104.5	104.5		104.5
1987	138.4	138.4		138.4
1988	59.1	59.1		59.1
1989	96.3	96.3		96.3
1990	75.0	75.0		75.0
1991	83.2	73.6	-9.6	83.2
1992	69.7	59.1	-10.6	69.7
1993	63.1	44.1	-19.0	44.1
1994	67.8	51.3	-16.5	56.1
1995	0.0	49.8	49.8	30.1
1996	0.0	44.4	44.4	41.7
1997	0.0	29.6	29.6	7.3
TOTAL	911.4	979.5	68.1	959.8

Table 4-17. WAM Program 1990 SAR Base-Year Budget

Budget Year	1990 Budget	Final SAR Budget	Difference	Revised Budget
1982	15.4	15.4		15.4
1983	18.7	18.7		18.7
1984	47.1	47.1		47.1
1985	74.5	74.5		74.5
1986	100.0	100.0		100.0
1987	126.7	126.7		126.7
1988	52.7	52.7		52.7
1989	81.9	81.9		81.9
1990	61.9	61.9		61.9
1991	66.2	58.5	-7.6	66.2
1992	53.9	45.7	-8.2	53.9
1993	47.8	33.4	-14.4	33.4
1994	50.5	38.2	-12.3	41.8
1995	0.0	36.4	36.4	22.0
1996	0.0	31.9	31.9	29.9
1997	0.0	21.0	21.0	5.2
TOTAL	797.3	844.0	46.7	831.3

The 1990 SAR budgets displayed in Table 4-16 (then-year dollars) and Table 4-17 (base-year dollars) shows $35.5M in then-year dollars curtailed from the program starting in 1993 and $123.8M in then-year dollars being reprogrammed into the program's out years. The 1990 SAR indicates the program slips three years when this curtailment occurs. The method underestimates the final budget by 2.01% in then-year dollars and 1.50% in base-year dollars. The model underestimates the final RDT&E budget in both applications.

Table 4-18. WAM Program Weibull Parameters

Program	Original					Revised				
	Year	Location	Shape	Scale	t_final	**Year**	Location	Shape	Scale	t_final
WAM Summary	1989	0.711592	2.062697	8.188366	15.75536	1989	0.711592	1.91584	9.373868	18.75536
WAM Summary	1990	1.204005	1.874529	7.421058	15.69646	1990	1.204005	1.721063	8.438306	18.69646

Table 4-18 above presents the Weibull parameters for the WAM program budget before the curtailment (Original) and when the curtailment occurred (Revised). The *location* parameter remains the same because the beginning of the program has not changed. The change in the *shape* parameter indicates the peak expenditure for this budget profile is proportionally earlier than originally programmed. The *scale* parameter growth is indicative of the overall schedule growth of the program. The t_{final}^{rev} parameter increases by the schedule growth.

Results

The methodology outlined in Chapter Three is applied to the programs meeting the criteria put forth at the beginning of this chapter. The final budget forecast using the methodology is compared with the final SAR containing RDT&E expenditures. All comparisons are done in then-year dollars and base-year dollars to assure the same baseline is being used.

Table 4-19. Method Accuracy Summery

Program	Year	Method Accuracy Then-Year	Model Accuracy Base-Year
MLRS-TGW	1985	-1.25%	-0.23%
ADCAP	1986	-5.24%	-3.99%
ADDS	1986	12.80%	10.52%
TRI-TAC	1983	6.74%	5.28%
WAM	1989	2.53%	2.45%
WAM	1990	-2.01%	-1.50%
Average Method Accuracy		**2.26%**	**2.09%**
Average Absolute Value		**5.10%**	**4.00%**

Table 4-19 presents all the programs and the method accuracy for each respective program. The first column is the program name. The second column is the year where funding was curtailed. The third column is the method's accuracy to predict the final budget as a percentage of the actual final RDT&E budget SAR in then-year dollars. The fourth column is the same comparison as the third column but base-year (constant) dollars are used. The bottom row is the average method accuracy error. Again, the closer the percentage is to zero, the better the method's accuracy to predict the final budget.

Discussion of Results. The model forecasts nearly all of the programs within +/- 7% of the final RDT&E budget SAR. The model forecasts all program's final RDT&E budget SAR within +/- 13%.

The method's average error is 2.26% over the actual then-year dollar budget. This indicates the model is a little conservative in its estimation of the final budget. The method's average error improves to an overestimation of 2.09% in base-year dollars. The low percentage of error is considered good and supports the conclusion that this methodology provides a good approximation of the effect of a budget curtailment. Generally, the estimated budget was higher than the actual budget for the year following the curtailment. Since authority for the curtailed funds is usually transferred from the military department to the Office of the Secretary of Defense (OSD), we are not surprised that programs that are curtailed are not fully funded in the next fiscal year. Why should the military departments submit money that has a high risk of being taken? As a result, most programs that are curtailed have several following years where the base-year dollar total of their program is below the constant dollar amount required to complete the program. The underfunding of R&D programs made us use the final SAR, which by definition is a fully funded program, as the requirement. In base-year dollars, our method is even more conservative. In conclusion, the result of this approach to forecast budgets for programs in the RDT&E phase when funding curtailment is considered successful.

Chapter Summary

This chapter provides the results of applying the methodology to historical program costs data. A brief description of each program meeting the criteria was given along with information about the funding curtailment observed. The percentage of error provides model forecasting accuracy of the final RDT&E budget. Finally, model accuracy for all programs is shown with discussion of those results.

Chapter Three outlines the proposed methodology. We define success as being able to have the model forecast what the final RDT&E budget would be. This chapter reports excellent results from six applications. The following chapter, Chapter Five, discusses the conclusions that are drawn based upon the analysis performed in this chapter.

V. Conclusion and Recommendations

Chapter Overview

Summary findings and conclusions of this research effort are presented in this chapter. First, a brief restatement of the findings from Chapters One, Two, and Three, and summary of the results from Chapter Four are provided. Next, general conclusions regarding this overall research effort are discussed. This discussion includes a comparison of the results of this research effort to previous efforts. Finally, recommendations of areas for future related research are presented

Synopsis

Chapter One introduces the problem of calculating the consequences of funding curtailment to a program in the R&D phase. In addition, it brings to light that the consequences most often associated with funding curtailment are program cost and schedule growth. Finally, the chapter concludes that the purpose of this research is to provide a quick and reasonably accurate estimation tool of funding impact when considering funding curtailment.

Chapter Two summarizes the findings of the literature review for this research effort. We discuss several cost models before explaining the capabilities of the Rayleigh model. Several cited papers show that the Rayleigh model has the capability to forecast project budgets. We identify the parameters needed to use the Rayleigh model and we provide information on how each parameter is derived. We describe the Weibull function as the means to model real program expenditures. The flexibility of the Weibull function provides a more accurate budget and expenditure model. The chapter concludes with a discussion of the impact outlay rates have on the budget profile.

Chapter Three outlines the methodology we use to develop budget profiles when considering funding curtailment. We explain the steps for converting then-year dollars to base-year dollars. The chapter outlines the nonlinear programming step we take to derive the Weibull parameters for the original and revised budgets. Finally, we explain how the revised budget is built using past years' budgets for years already past, the planned budget amount for the year of the curtailment, and the estimated budget based on Weibull forecast amounts for future years.

Chapter Four provides the results of the methodology applied to real historical programs. We explain the criteria used in selecting the programs along with the rationale for each criterion. The tables show the curtailment of each selected program. We explain the Weibull parameters of the selected programs. We present the definition of success as the ability of the methodology to forecast the same amount as the final RDT&E SAR budget amount. All selected program budget forecasts were within 6% of the actual budget amount. The results are presented as a percentage of how accurately the method is able to forecast the final budget amount.

Conclusion

We conclude the proposed methodology does provide a very accurate and quick ability to forecast revised R&D budgets when funds are delayed. Thus, we achieved our research objective. The method forecasts nearly all of the programs within +/-7% of the final RDT&E budget SAR with an average method accuracy error of 2.26% in then-year dollars and 2.09% in base-year dollars. This indicates the model is a little conservative in its estimation of the final budget. We consider the low percentage of error good, which supports the conclusion that this methodology provides a good approximation of the effect of a budget curtailment.

Areas for Future Related Research

Contributing Factors. During the analysis of the modeled programs, we tracked several characteristics of the program. For example, for each modeled program we noted the percent of the curtailment when compared to the overall budget, how far along the program was before the curtailment occurred, and the Weibull parameters for each program among other characteristics.

Because of the small number of programs modeled in establishing this approach, insufficient data exists to statistically distinguish factors that can be identified as contributing to or derogating from the model's accuracy. Future research could be conducted in search of identifiable factors that would provide the reader, and ultimately the user, a better picture of what contributes to the model's accuracy or inaccuracy as the case may be.

Other Cost Models. This research effort uses the Rayleigh Model and its parent, the Weibull Model, as the means to model each program. Chapter Two highlights other cost models available in the literature. Although we chose not to use these other models for reasons stated, there is value in applying those cost models to learn the accuracy of providing an accurate forecast of future budgets when considering funding curtailment.

Appendix A. An Alternative Approach

Appendix Note

There are different methods available to address the research question. During the research effort, we pursued a path we thought would provide the analytic methodology we sought. After several weeks of research into this area, we presented this methodology to Dr. Burke (OSD PA&E), our sponsor. He informed us that one of the assumptions we were using was incorrect. Work accomplished does not provide benefit to the project if resources are not available to use the work results. Needless to say, we pursued another methodology.

We have included this alternative methodology to aid the reader in understanding why this approach, at this time, would not be appropriate. The mathematical basis for the methodology is sound and is provided.

Appendix Overview

This appendix follows the reasoning laid out in Chapter Two as the basis for using the Rayleigh function to model expenditure profiles. Figure 2-1 presents a cumulative expenditure profile for a program. Figure 3-1 presents cumulative expenditure profiles for a program with a funding curtailment. The parameters needed for this methodology are the same as presented in Chapter Two.

The methodology diverges from the methodology in Chapter Three at the point of curtailment.

Once the final time has been determined, the next step is to derive an estimate of the funding profile needed that will follow the Rayleigh probability density function. The starting point is the period where actual time and expenditures have occurred and ends at the revised final time.

The Rayleigh model has been presented as a function of time. The model states how much money is spent in the Rayleigh cumulative expenditure function distribution and the rate of expenditures in the Rayleigh probability expenditure distribution at a given time. Gallagher and Lee provide key insight on how the research question can be answered (Gallagher, 1996:52-53). In order to solve the thesis question, the focus needs to change to look at the rate of expenditure at a specific earned value—amount spent on a project. In essence, at what rate should the project be spending now that the project is has spent a specific amount?

We know from the Rayleigh model literature that during the first phase of the project the rate of spending increases up to a certain peak point, t_{peak}. After that point is reached, the rate of spending decreases as the proportion of problems remaining to be solved decreases.

This expenditure rate is assumed to be continuous, which means that the expenditure rate the project had one day will be similar to the expenditure rate the next day. The expenditure rate increases prior to its peak and decreases after the peak is attained. In other words, the expenditure rate is not disjointed. The expenditure rate will not be drastically different in contiguous time periods. This is evidenced by the smooth line/curve of the Rayleigh distributions.

Up to this point, we understand that the expenditure profile follows the Rayleigh function. We also know that if funding is changed, that a new Rayleigh function can be applied to model this new change. This new Rayleigh function will project a new project completion time, t_{final}^{rev}. Let v equal the amount of earned value—the amount already spent on the project. The derivative of v with respect to time provides the expenditures at certain point in time, dv/dt. The change in funding usually occurs after an expenditure of funds has already occurred, v_i, at some point in time, t_i.

We present dv/dt as a function of time. If we derive dv/dt as a function of cumulative expenditures (earned value), v, then the appropriate rate of expenditure can be projected when the amount of earned value is known.

The task is to fit a Rayleigh model that starts at point (t_i, v_i) and terminates at point (t_{final}^{rev}, d)—the same final point of the new Rayleigh function. This task is illustrated in Figure A-1 and the following simple example scenario.

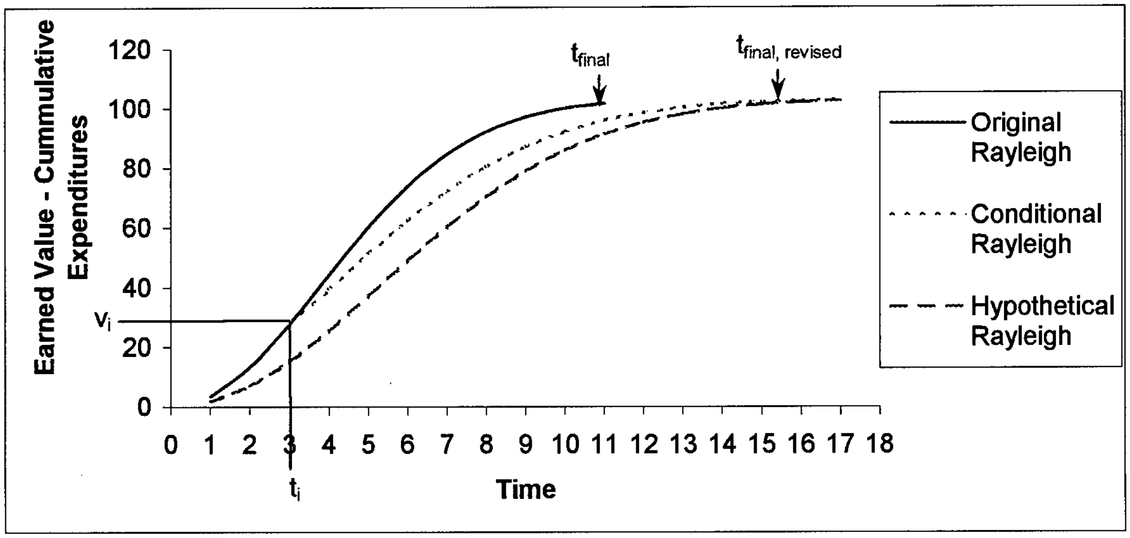

Figure A-1. Conditional Rayleigh Incorporating Time and Expenditures

For the example scenario, assume Figure A-1 is the graphical representative model of a current program. The solid line represents the original cumulative expenditures of the sample program. This line follows the Rayleigh cumulative distribution. At Time 3, the mangers wish to reduce the rate at which resources are consumed by this program. Therefore, the completion of the program is delayed from t_{final}^{org} to t_{final}^{rev}.

The new completion point using the Rayleigh model will have a different shaping parameter a than the original Rayleigh model. If the program had started with this new funding profile, assume the Rayleigh cumulative density function would have been similar to the dashed 'Hypothetical Rayleigh' line in Figure A-1.

Since funds have been expended to point v_i and three time periods have passed, t_i, the 'Hypothetical Rayleigh' line does not take into consideration the time and money actually used. Therefore, a new equation based on the Rayleigh model must be developed that takes into consideration the money and time already put forth into the program's R&D effort. The equation should start at point (t_i, v_i) and end at t_{final}^{rev}. The dotted line 'Conditional Rayleigh' in Figure A-1 represents the equation of that line.

Up to this point, the goal of calculating the Condition Rayleigh provides us with our objective. Gallagher and Lee provide adaptable mathematical formulation techniques that will suite our purpose. It is assumed the reader is knowledgeable of basic algebra and calculus. For an in depth analysis of the equation, the reader is referred to Gallagher, 1996. The next step is to derive an equation that will allow us to 'draw' a Rayleigh curve from point (t_i, v_i) and end at t_{final}^{rev}.

Let *v* equal the amount of earned value in constant dollars—the amount already spent on the project, the derivative of *v* with respect to time will provide the needed expenditure rate. Funding cannot be negative—once a program spends it, it cannot be recovered. As such, the derivative of any function of *v* will always be positive.

Assume the equation *F(v)* needs to be stated in how much spent per unit of time. As stated before, let *v* equal the amount of earned value—the amount already spent on the project, the derivative of *v* with respect to time provides the needed expenditure rate. This can be shown as

$$F(v) = \frac{dv}{dt}. \tag{18}$$

Assume that a function, in this case *P(v)* can be related to time. In other words, given a certain time point, the result from equation *P(v)* can be derived. In equation,

$$P(v) = t. \tag{19}$$

To relate (18) with (19), (19) must be solved for *dv/dt.* The derivative of (19) with respect to time would be

$$P'(v)dv = dt.$$

Divide both sides by *dt*

$$\frac{P'(v)dv}{dt} = 1.$$

Solving for *dv/dt* gives

$$\frac{dv}{dt} = \frac{1}{P'(v)}. \tag{20}$$

By substituting (18) into (20), we derive

$$F(v) = \frac{dv}{dt} = \frac{1}{P'(v)}. \qquad (21)$$

through algebraic manipulation the above equation can be stated

$$P'(v) = \frac{1}{F(v)}.$$

F(v) is positive because the rate of expenditures, *dv/dt,* cannot be negative. Therefore, *P'(v)* is always positive, and *P(v)* increases continually. This means that for each point of cumulative expenditures a single corresponding time exists. In other words, for each earned value point there is exactly one corresponding point in time. This characteristic ensures the inverse of the function *P(v)* exists. Stated in a formula

$$P^{-1}(t) = v.$$

These above rules hold for any function. The purpose of our thesis question concerns the Rayleigh function. To put the Rayleigh function in the context described here, the Rayleigh function can be described as (adapted from (3))

$$v = d\left(1 - \left(e^{-at^2}\right)\right)$$

We solve for *t* to get *P(v)* by first dividing out the *d* parameter resulting in

$$\frac{v}{d} = 1 - e^{-at^2}.$$

Bringing *e* to the left side of the equation to make it positive results in

$$e^{-at^2} = 1 - \frac{v}{d}.$$

Taking the natural logarithm to remove *e* results in

$$-at^2 = \ln\left(1 - \tfrac{v}{d}\right).$$

Dividing through by *a* results in

$$t^2 = \frac{-1}{a}\ln\left(1-\tfrac{v}{d}\right).$$

Taking the square root of both sides to remove the exponential of the *t* parameter results in

$$t = \left(-\tfrac{1}{a}\ln\left(1-\tfrac{v}{d}\right)\right)^{\frac{1}{2}}. \qquad (22)$$

From (22), we now have *P(v)* for the Rayleigh function (3). Equation (7) provides the mathematical relation ship between *P(v)* and *F(v)*. Therefore, to get *F(v)* from the *P(v)* the derivative of *P(v)* (22) must be taken. The derivative is

$$dt = \tfrac{1}{2}\left[-\tfrac{1}{a}\ln\left(1-\tfrac{v}{d}\right)\right]^{-\frac{1}{2}}\left(-\tfrac{1}{a}\right)\frac{1}{1-\frac{v}{d}}\left(-\tfrac{1}{d}\right)dv.$$

Combining like terms results in

$$dt = \frac{v}{2ad}\left(\frac{1}{1-\frac{v}{d}}\right)\left[-\tfrac{1}{a}\ln\left(1-\tfrac{v}{d}\right)\right]^{-\frac{1}{2}}dv.$$

By inverting the preceding equation, we will obtain *dv/dt* which per (18) equals *F(v)*

$$F(v) = \frac{dv}{dt} = 2ad\left(1-\tfrac{v}{d}\right)\left[-\tfrac{1}{a}\ln\left(1-\tfrac{v}{d}\right)\right]^{\frac{1}{2}}.$$

As stated before, all expenditures are positive. Only at the inception of the program are they zero. Stated in formula notation is

$$v(0) = 0.$$

At any point in time, t_i, the expenditure rate will be v_i,

$$v(t_i) = v_i.$$

Solving (18) with initial conditions of $v(t_i) = v_i$ we arrive at

$$v(t) = P^{-1}(t-t_i) + P(v_i)$$

Substituting this back into the Rayleigh function results in

$$= d\left(1 - e^{-a(t+t_i)+P(v_i)^2}\right)$$

By substituting in the formula of *P(v)* above, the final formula is

$$v(t) = d\left(1 - e^{-a\left((t-t_i)+\left[\frac{-1}{a}\ln\left(1-\frac{v}{d}\right)\right]^{\frac{1}{2}}\right)^2}\right) \qquad (23)$$

Now that we have the needed equation, the final task is to use (23) with the relevant time and earned value numbers to model R&D programs that have had changes to their funding and/or expenditure profiles. The final step to accomplish this is to extrapolate the budget profile through the outlay and appropriation filters from a required outlay profile.

Appendix Summary

In summary, the methodology we propose can be broken down into the following steps: A new final time is determined, t_{final}^{rev}. This final time provides the appropriate a parameter to use in (23). An outlay profile will be developed using (23) and from this outlay profile, the same approach may be applied to determine the budget.

Appendix B. Inflation Indices for Model Programs

Appendix Overview

Converting then-year dollars to base-year dollar requires inflation factors. This appendix contains the raw inflation indices for the selected programs. We use these factors to remove inflation from yearly expenditures. Bolded numbers are the base year for the particular program.

The final table contains weighted inflation factors. We use these weighted factors when removing inflation from a budget.

Program Inflation Indices

MLRS-TGW Program. The MLRS-TGW program is an Army Missile program. The base year for this program is 1984.

Table B-1. Inflation Index for the MLRS-TGW Program

Year	Index
1980	0.752
1981	0.841
1982	0.918
1983	0.963
1984	**1.000**
1985	1.034
1986	1.063
1987	1.092
1988	1.124
1989	1.172
1990	1.218
1991	1.271
1992	1.306
1993	1.342
1994	1.369
1995	1.395
1996	1.422
1997	1.452
1998	1.463
1999	1.474
2000	1.489
2001	1.511
2002	1.534
2003	1.557
2004	1.588
2005	1.620
2006	1.652
2007	1.685
2008	1.719
2009	1.753
2010	1.788

The ADCAP Program. The ADCAP program is a Navy Munitions program. The base year for this program is 1989.

Table B-2. Inflation Index for the ADCAP Program

Year	Index
1979	0.586
1980	0.641
1981	0.718
1982	0.784
1983	0.822
1984	0.854
1985	0.883
1986	0.907
1987	0.932
1988	0.960
1989	**1.000**
1990	1.040
1991	1.085
1992	1.115
1993	1.145
1994	1.168
1995	1.190
1996	1.214
1997	1.240
1998	1.248
1999	1.258
2000	1.271
2001	1.290
2002	1.309
2003	1.329
2004	1.355
2005	1.383
2006	1.410
2007	1.438
2008	1.467
2009	1.497

The ADDS Program. The ADDS program is an Army Electronic program. The base year for this program is 1983.

Table B-3. Inflation Index for the ADDS Program

Year	Index
1981	0.873
1982	0.953
1983	**1.000**
1984	1.038
1985	1.073
1986	1.103
1987	1.133
1988	1.167
1989	1.216
1990	1.265
1991	1.319
1992	1.356
1993	1.393
1994	1.421
1995	1.448
1996	1.477
1997	1.508
1998	1.518
1999	1.530
2000	1.546
2001	1.569
2002	1.592
2003	1.616
2004	1.648
2005	1.681
2006	1.715
2007	1.749
2008	1.784
2009	1.820
2010	1.856
2011	1.894

<u>The TRI-TAC Program</u>. The TRI-TAC program is an Air Force Electronic program. The base year for this program is 1991.

Table B-4. Inflation Index for the TRI-TAC Program

Year	Index
1992	1.028
1993	1.056
1994	1.077
1995	1.097
1996	1.119
1997	1.143
1998	1.151
1999	1.160
2000	1.172
2001	1.189
2002	1.207
2003	1.225
2004	1.250
2005	1.275
2006	1.300
2007	1.326
2008	1.353
2009	1.380
2010	1.407
2011	1.435
2012	1.464
2013	1.493
2014	1.523
2015	1.554
2016	1.585
2017	1.617
2018	1.649
2019	1.682
2020	1.715
2021	1.750
2022	1.785

The WAM Program. The WAM Program is an Air Force Electronic program. The base year for this program is 1982.

Table B-5. Inflation Index for the WAM Program

Year	Index
1982	**1.000**
1983	1.049
1984	1.089
1985	1.126
1986	1.157
1987	1.189
1988	1.224
1989	1.276
1990	1.327
1991	1.384
1992	1.423
1993	1.461
1994	1.490
1995	1.519
1996	1.549
1997	1.581
1998	1.592
1999	1.605
2000	1.621
2001	1.646
2002	1.670
2003	1.695
2004	1.729
2005	1.764
2006	1.799
2007	1.835
2008	1.872
2009	1.909
2010	1.947
2011	1.986
2012	2.026

Weighted Inflation Factors

The weighted inflation factors are used to remove inflation from budget figures. This table is used for the base-year comparisons.

Table B-6. Weighted Inflation Index for Budgets

Year	Index
1970	0.373
1971	0.392
1972	0.410
1973	0.433
1974	0.467
1975	0.513
1976	0.557
1977	0.590
1978	0.639
1979	0.690
1980	0.767
1981	0.849
1982	0.908
1983	0.950
1984	0.987
1985	**1.020**
1986	1.045
1987	1.092
1988	1.121
1989	1.176
1990	1.212
1991	1.257
1992	1.294
1993	1.321
1994	1.344
1995	1.369
1996	1.394
1997	1.412
1998	1.421
1999	1.434
2000	1.452

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14. ABSTRACT

This research develops an analytical technique to estimate the impact of funding curtailment on an R&D program. The method quickly produces a revised budget by year for an on-going R&D program when funding in one year is reduced. We assume program requirements remain unchanged. The program duration may be unchanged or "stretched" to a later completion date. We use the Rayleigh and Weibull functions to model expenditure profiles, which forms basis of the analytical approach. Our proposed methodology accounts for budget outlay rates and inflation. We validate the proposed analytical technique using historical cost data from several programs.

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